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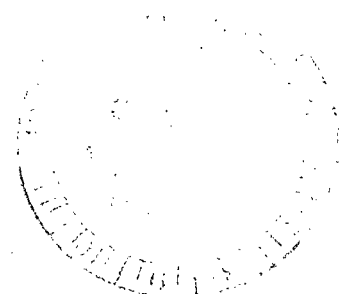


**A Numerical Solution  
of the Navier-Stokes Equations  
for Chemically Nonequilibrium,  
Merged Stagnation Shock Layers  
on Spheres and Two-Dimensional  
Cylinders in Air**

**Kenneth D. Johnston and William L. Hendricks**

**MAY 1978**

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## A Numerical Solution of the Navier-Stokes Equations for Chemically Nonequilibrium, Merged Stagnation Shock Layers on Spheres and Two-Dimensional Cylinders in Air

Kenneth D. Johnston

*George C. Marshall Space Flight Center*

*Marshall Space Flight Center, Alabama*

and

William L. Hendricks

*Lockheed Huntsville Research and Engineering Center*

*Huntsville, Alabama*



National Aeronautics  
and Space Administration

**Scientific and Technical  
Information Office**

1978

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## DEFINITION OF SYMBOLS

<u>Symbol</u>	<u>Definition</u>
$\bar{C}_p$	specific heat at constant pressure
$\bar{C}_v$	specific heat at constant volume
$C_H$	heat transfer coefficient, $\bar{q}_w / \frac{1}{2} \bar{\rho}_\infty \bar{V}_\infty^2$
$D_{ij}$	binary diffusion coefficient for species pair i and j
$\bar{e}_i$	specific internal energy of species i
$\vec{f}_i$	body force on species i per unit mass of species i
$\bar{h}$	specific enthalpy of mixture
$\bar{h}_i$	specific enthalpy of species i
$\bar{k}$	coefficient of thermal conductivity
$k_{fj}$	forward reaction rate for jth reaction
$k_{rj}$	reverse reaction rate for jth reaction
$K_{n\infty}$	freestream Knudsen number, $\bar{\lambda}_\infty / \bar{r}_b$
$K^2$	hypersonic similarity parameter [see equation (62)]
$Le_{ij}$	Lewis number of species pair i and j, $\bar{\rho} D_{ij} \bar{C}_p / \bar{k}$
$M_\infty$	freestream Mach number
$M$	indicates a catalyst
$NS$	number of species in gas mixture
$n$	dimensionless radial distance from the body, $(\bar{r} - \bar{r}_b) / \bar{r}_b$
$n_\infty$	dimensionless radial distance from body to freestream edge (shock layer thickness)

## DEFINITION OF SYMBOLS (Continued)

<u>Symbol</u>	<u>Definition</u>
$\bar{p}$	pressure
Pr	Prandtl number, $\bar{C}_p \bar{\mu} / \bar{k}$
$\vec{q}$	heat flux vector
$\bar{q}_w$	heat flux to wall
$\bar{r}$	radial distance from body center (see Fig. 1)
$\bar{r}_b$	radius of body
$\bar{R}_i$	gas constant of species i
$\bar{R}$	gas constant of mixture
$\bar{R}$	universal gas constant
$Re_{0\infty}$	freestream stagnation Reynolds number, $\bar{\rho}_\infty \bar{V}_\infty \bar{r}_b / \bar{\mu}_{0\infty}$
$\bar{S}_{ij}$	collision cross section for particle i with particle j
$Sc_{ij}$	Schmidt number for species pair i and j, $\bar{\mu} / \bar{\rho} \bar{D}_{ij}$
$\bar{T}$	temperature
$\bar{T}_{0\infty}$	freestream stagnation temperature
$\bar{t}$	time
$\bar{u}$	velocity component parallel to body surface (see Fig. 1)
$\bar{v}$	velocity component normal to body surface (see Fig. 1)
$\bar{v}$	specific volume of gas mixture
$\vec{v}$	velocity vector



## DEFINITION OF SYMBOLS (Continued)

<u>Symbol</u>	<u>Definition</u>
$\bar{V}_\infty$	freestream speed
$\vec{V}_i$	diffusion velocity vector of species i
$W_i$	molecular weight of species i
$W$	equivalent molecular weight of mixture
$\dot{\bar{w}}_i$	net mass production rate of species i per unit volume
$Y_i$	mass fraction of species i
$\gamma$	ratio of specific heats, $C_p/C_v$
$\gamma_i$	recombination coefficient for species i
$\eta$	dimensionless radial distance from body surface [see Fig. 1 and equation (24)]
$\Theta_{vi}$	characteristic temperature for vibration of diatomic species i
$\theta$	circumferential angle (see Fig. 1)
$\bar{\lambda}_i$	mean free path of species i
$\bar{\lambda}_\infty$	freestream mean free path
$\bar{\mu}$	coefficient of absolute viscosity
$\bar{\mu}_{SU}$	Sutherland coefficient of absolute viscosity
$\bar{\mu}_{0\infty}$	coefficient of absolute viscosity at $\bar{T}_{0\infty}$
$\bar{\rho}$	density of mixture
$\sigma$	molecule-surface accommodation coefficient

## DEFINITION OF SYMBOLS (Concluded)

<u>Symbol</u>	<u>Definition</u>
$\vec{\tau}$	viscous stress tensor
$\bar{\Phi}$	viscous dissipation function

### Subscripts

i, j	species indices
0 (zero)	stagnation condition
s	edge of Knudsen layer, slip
sh	conditions behind normal shock
w	wall
$\infty$	freestream property

### Superscript

( $\bar{\phantom{x}}$ )	dimensional quantity
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## TECHNICAL PAPER

# A NUMERICAL SOLUTION OF THE NAVIER-STOKES EQUATIONS FOR CHEMICALLY NONEQUILIBRIUM, MERGED STAGNATION SHOCK LAYERS ON SPHERES AND TWO-DIMENSIONAL CYLINDERS IN AIR

## SUMMARY

The complete Navier-Stokes equations are solved along the stagnation streamline in merged stagnation shock layers on spheres and two-dimensional cylinders using an iterative finite-difference numerical procedure known as the accelerated successive replacement method. The fluid medium is chemically reacting air consisting of seven species. Velocity components, thermodynamic properties, species mass fractions, and wall heat transfer rates are computed. This report is intended as an explanation of the method and as a user's manual for the computer program.

## I. INTRODUCTION

An aerospace vehicle ascending or descending through the Earth's atmosphere traverses several flow regimes from the continuum boundary layer regime at low altitudes, through the transitional regime at intermediate altitudes, to the free molecular regime at very high altitudes. The character of the flow field changes drastically from the boundary layer regime to the free molecular regime, and no single computational approach is valid throughout this range. The broad transitional regime may be divided into several sub-regimes as suggested by Hayes and Probstein [1]. Consider the typical spherical nose of an aerospace vehicle. In the boundary layer regime viscous effects are primarily confined to a thin boundary layer, the bow shock can be treated as a discontinuity, and a region of inviscid flow exists between the shock and the boundary layer. However with increasing altitude, the shock wave and boundary layer thicken and eventually merge into a single viscous layer called the shock layer. The flow regime in which this occurs is called the fully merged shock layer regime which is the condition treated in this report.

At the great speed that a vehicle reenters the atmosphere, the temperature near the body becomes extremely high, especially in the stagnation region. Therefore, the air in the shock layer dissociates and ionizes. For an accurate description of the flow field, one must account for these real gas effects. Also an accurate estimate of the ionization level is needed for radio communication purposes. Therefore, a flow field model including finite rate chemistry is required.

This report describes a method for computing flow properties along the stagnation streamlines of a sphere and a circular cylinder transverse to the flow. Heat transfer rates are computed at the body surface. Although this method is limited to the stagnation region, it still provides valuable design information because maximum heating rates usually occur at the nose. The computational method was developed by Jain and Adimurthy [2] for an ideal gas. The method uses the full Navier-Stokes equations to describe the flow in the entire shock layer from the surface to the freestream. The boundary conditions at the wall are provided by slip velocity and temperature jump equations. Using the concept of local similarity, the governing equations are reduced to a system of nonlinear, coupled ordinary differential equations. Numerical solutions are obtained for points on the stagnation streamline using an iterative finite-difference procedure known as the accelerated successive replacement method. The applicability of this approach and the failure of thin-layer theories for the merged shock layer regime is discussed in Reference 2. Nonequilibrium chemical reactions were included in this method by Kumar and Jain [3] using an air model with seven species and six reactions. Hendricks [4] developed surface slip velocity and temperature jump equations for a multi-component gas, including the effects of wall catalysis, to use with this model. Additional modifications have been made in this report, principally by including the two-dimensional cylindrical geometry and using a multi-component gas model to compute viscosity.

## II. ANALYSIS

### A. Formulation of the Problem

1. Approach. In the present analysis the full Navier-Stokes equations, with nonequilibrium chemistry, are solved through the merged stagnation shock layer from the freestream to the body. The slip conditions at the gas-wall interface include the effect of wall catalysis and a multicomponent, nonequilibrium gas flow.

The thin shock layer assumption is not made in the present analysis. The full Navier-Stokes equations with chemically-reacting, nonequilibrium air are solved through the merged shock and boundary layer. This allows the shock wave to develop within the computational domain. A seven species air model is used. The species considered are  $N_2$ ,  $O_2$ ,  $NO$ ,  $N$ ,  $O$ ,  $NO^+$ , and  $e^-$ . For air dissociation and ionization, the rate expressions recommended by Wray [5] are adopted. Prandtl number,  $Pr$ , and Lewis number,  $Le_{ij}$ , are taken to be 0.75 and 1.4, respectively, for the cases computed in this report. The viscosity of dissociated and ionized air is obtained from a simple summation formula for a mixture of hard spherical molecules using Hansen's collision cross sections [6].

Solutions are obtained by using the local similarity concept to reduce the governing equations to a set of nonlinear, coupled, ordinary differential equations. This set of equations is integrated using a finite difference method known as the accelerated successive replacement method. It is important to note that the first order local similarity assumption is a good approximation near the stagnation streamline, at least for  $Re_{sh} \geq 10$  [7]. Reference 7 gives a thorough discussion of local similarity.

2. Governing Equations. The nonlinear, coupled ordinary differential equations governing the flow of a multicomponent gas near the stagnation streamlines of spheres and two-dimensional cylinders are presented. The coordinate system employed is shown in Figure 1a.

a. Basic Equations. The basic conservation equations and the ideal gas equation of state for a multicomponent, reacting gas mixture are as follows [8]:

Global Continuity (of all species):

$$\frac{\partial \bar{\rho}}{\partial \bar{t}} + \bar{\nabla} \cdot (\bar{\rho} \bar{\vec{v}}) = 0 \quad . \quad (1)$$

Species Continuity:

$$\bar{\rho} \frac{D Y_i}{D \bar{t}} = \dot{\bar{w}}_i - \bar{\nabla} \cdot (\bar{\rho} Y_i \bar{\vec{v}}_i) \quad i = 1, \dots, NS \quad , \quad (2)$$

where  $NS$  = number of species in mixture (no summation on repeated indices).

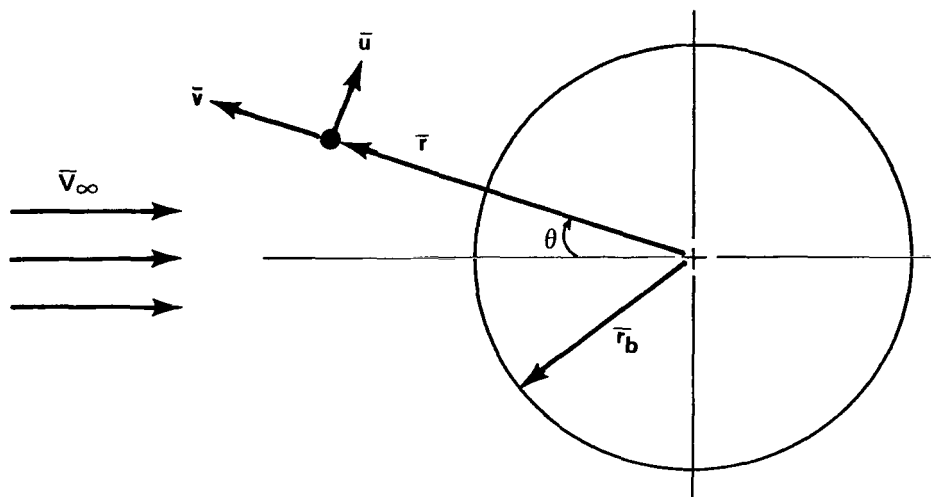


Figure 1a. Dimensional coordinate system.

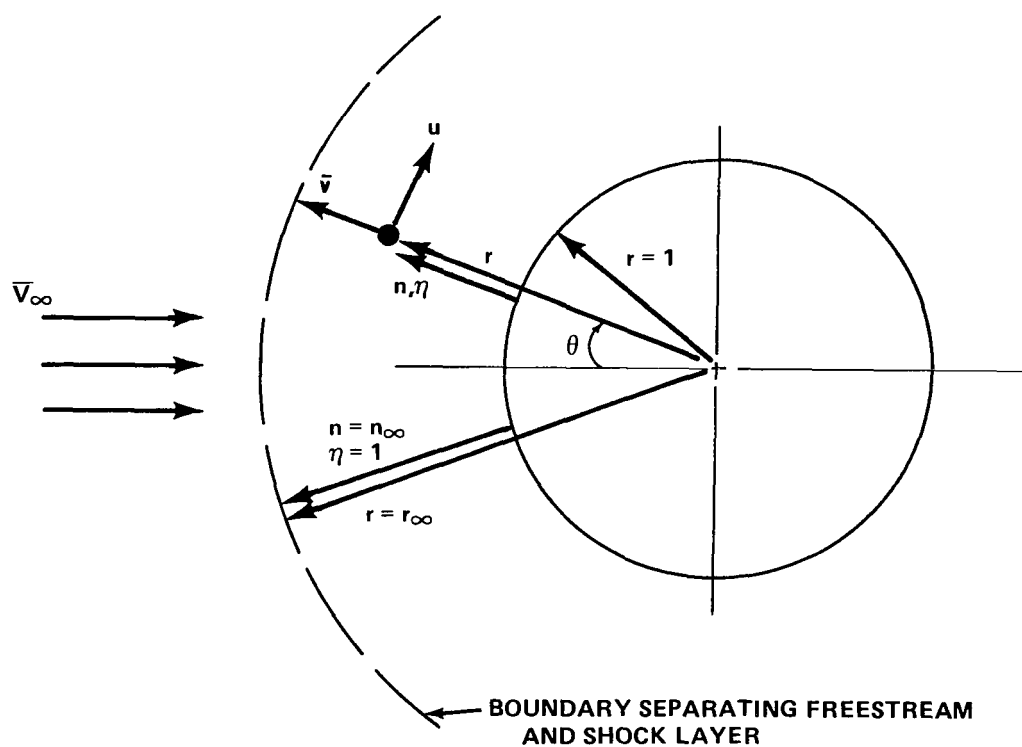


Figure 1b. Dimensionless coordinate system.

Figure 1. Coordinate systems.

Momentum:

$$\bar{\rho} \frac{D \vec{v}}{Dt} = - \vec{\nabla} \bar{P} - \vec{\nabla} \cdot \vec{\tau} + \bar{\rho} \sum_{i=1}^{NS} Y_i \vec{f}_i \quad . \quad (3)$$

The second order tensor,  $\vec{\tau}$ , is the viscous stress tensor and  $\vec{f}_i$  is the body force per unit mass of species  $i$ .

Enthalpy:

$$\bar{\rho} \frac{D \bar{h}}{Dt} = \frac{D \bar{P}}{Dt} + \bar{\Phi} - \vec{\nabla} \cdot \vec{q} \quad . \quad (4)$$

The quantity  $\bar{\Phi}$  is the viscous dissipation function and  $\vec{q}$  is the heat flux vector.

State:

$$\bar{P} = \bar{\rho} \bar{R} \bar{T} \sum_{i=1}^{NS} \frac{Y_i}{W_i} \quad . \quad (5)$$

b. Nondimensional Equations. The basic equations are put in non-dimensional form by introducing dimensionless variables as follows:

$$u = \bar{u} / \bar{V}_{\infty} \quad ,$$

$$v = \bar{v} / \bar{V}_{\infty} \quad ,$$

$$\rho = \bar{\rho} / \bar{\rho}_{\infty} \quad ,$$

$$T = \bar{T} / \bar{T}_{0\infty} \quad ,$$

$$P = \bar{P}/\bar{\rho}_\infty \bar{V}_\infty^2 \quad ,$$

$$h = \bar{h}/\bar{V}_\infty^2 \quad ,$$

$$\mu = \bar{\mu}/\bar{\mu}_{0\infty} \quad ,$$

$$r = \bar{r}/\bar{r}_b \quad ,$$

$$\dot{w}_i = \bar{\dot{w}}_i/(\bar{\rho}_\infty \bar{V}_\infty/\bar{r}_b) \quad ,$$

where  $\bar{r}_b$  is the radius of the body,  $\bar{T}_{0\infty}$  is the freestream stagnation temperature, and  $\bar{\mu}_{0\infty}$  is the coefficient of absolute viscosity evaluated at  $\bar{T}_{0\infty}$ . The nondimensional similarity parameters

$$Re_{0\infty} = \bar{\rho}_\infty \bar{V}_\infty \bar{r}_b/\bar{\mu}_{0\infty} \quad ,$$

$$Pr = \bar{C}_p \bar{\mu}/\bar{k} \quad ,$$

$$Sc_{ij} = \bar{\mu}/\bar{\rho} \bar{D}_{ij} \quad ,$$

and

$$Le_{ij} = \bar{k}/\bar{\rho} \bar{C}_p \bar{D}_{ij}$$

are also introduced. The basic equations are then simplified by assuming steady flow and Newtonian fluid, neglecting body forces, viscous diffusion stresses, thermal radiation, and thermal diffusion, and using Fick's Law of Diffusion. The equations are presented in cylindrical and spherical coordinates below.



Cylindrical Coordinates:

Global Continuity:

$$\frac{\partial}{\partial r} (\rho r v) + \frac{\partial}{\partial \theta} (\rho u) = 0 \quad . \quad (6)$$

Species Continuity:

$$\rho \left( v \frac{\partial Y_i}{\partial r} + \frac{u}{r} \frac{\partial Y_i}{\partial \theta} \right) = \dot{w}_i + \frac{1}{\text{Re}_{0\infty}} \frac{1}{r} \left[ \frac{\partial}{\partial r} \left( \frac{\mu r}{\text{Sc}} \frac{\partial Y_i}{\partial r} \right) + \frac{\partial}{\partial \theta} \left( \frac{\mu}{\text{Sc} r} \frac{\partial Y_i}{\partial \theta} \right) \right] .$$

(7)

$i = 1, \dots, \text{NS}$

Transverse Momentum ( $\theta$  Direction):

$$\rho \left( v \frac{\partial u}{\partial r} + \frac{u}{r} \frac{\partial u}{\partial \theta} + \frac{vu}{r} \right) = - \frac{1}{r} \frac{\partial P}{\partial \theta} + \frac{1}{\text{Re}_{0\infty}} \left[ \frac{1}{r^2} \frac{\partial}{\partial r} \left\{ r^2 \mu \left[ r \frac{\partial}{\partial r} \left( \frac{u}{r} \right) + \frac{1}{r} \frac{\partial v}{\partial \theta} \right] \right\} \right. \\ \left. + \frac{1}{r} \frac{\partial}{\partial \theta} \left( \mu \left\{ 2 \left( \frac{1}{r} \frac{\partial u}{\partial \theta} + \frac{v}{r} \right) - \frac{2}{3} \left[ \frac{1}{r} \frac{\partial}{\partial r} (rv) + \frac{1}{r} \frac{\partial u}{\partial \theta} \right] \right\} \right) \right] . \quad (8)$$

Radial Momentum ( $r$  Direction):

$$\rho \left( v \frac{\partial v}{\partial r} + \frac{u}{r} \frac{\partial v}{\partial \theta} - \frac{u^2}{r} \right) = - \frac{\partial P}{\partial r} + \frac{1}{\text{Re}_{0\infty}} \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \mu \left\{ 2 \frac{\partial v}{\partial r} - \frac{2}{3} \left[ \frac{1}{r} \frac{\partial (rv)}{\partial r} + \frac{1}{r} \frac{\partial u}{\partial \theta} \right] \right\} \right) \right. \\ \left. + \frac{1}{r} \frac{\partial}{\partial \theta} \left\{ \mu \left[ r \frac{\partial}{\partial r} \left( \frac{u}{r} \right) + \frac{1}{r} \frac{\partial v}{\partial \theta} \right] \right\} - \frac{1}{r} \left( \mu \left\{ 2 \left( \frac{1}{r} \frac{\partial u}{\partial \theta} + \frac{v}{r} \right) \right. \right. \right. \\ \left. \left. \left. - \frac{2}{3} \left[ \frac{1}{r} \frac{\partial}{\partial r} (rv) + \frac{1}{r} \frac{\partial u}{\partial \theta} \right] \right\} \right) \right] . \quad (9)$$

Enthalpy:

$$\begin{aligned}
\rho \left( v \frac{\partial h}{\partial r} + \frac{u}{r} \frac{\partial h}{\partial \theta} \right) = & v \frac{\partial P}{\partial r} + \frac{u}{r} \frac{\partial P}{\partial \theta} + \frac{\mu}{\text{Re}_{0\infty}} \left[ 2 \left\{ \left( \frac{\partial v}{\partial r} \right)^2 + \left( \frac{1}{r} \frac{\partial u}{\partial \theta} + \frac{v}{r} \right)^2 \right\} \right. \\
& + \left. \left( \frac{1}{r} \frac{\partial v}{\partial \theta} + \frac{\partial u}{\partial r} - \frac{u}{r} \right)^2 \right] - \frac{2}{3} \frac{\mu}{\text{Re}_{0\infty}} \left[ \frac{\partial v}{\partial r} + \frac{1}{r} \frac{\partial u}{\partial \theta} + \frac{v}{r} \right]^2 \\
& + \frac{1}{r \text{Pr} \text{Re}_{0\infty}} \left\{ \frac{\partial}{\partial r} \left( r \mu \frac{\partial h}{\partial r} \right) + \frac{\partial}{\partial \theta} \left( \frac{\mu}{r} \frac{\partial h}{\partial \theta} \right) \right. \\
& + (\text{Le} - 1) \frac{\partial}{\partial r} \left[ \sum_{i=1}^{\text{NS}} r \mu h_i \frac{\partial Y_i}{\partial r} \right] + (\text{Le} - 1) \frac{\partial}{\partial \theta} \left[ \sum_{i=1}^{\text{NS}} \frac{\mu h_i}{r} \frac{\partial Y_i}{\partial \theta} \right] \left. \right\} .
\end{aligned} \tag{10}$$

State:

$$P = \rho T \frac{\bar{R} \bar{T}_{0\infty}}{\bar{V}_{\infty}^2} \sum_{i=1}^{\text{NS}} \frac{Y_i}{W_i} . \tag{11}$$

Spherical Coordinates:

Global Continuity:

$$\frac{1}{r} \frac{\partial}{\partial r} (\rho r^2 v) + \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} (\rho u \sin \theta) = 0 . \tag{12}$$

Species Continuity:

$$\rho \left( v \frac{\partial Y_i}{\partial r} + \frac{u}{r} \frac{\partial Y_i}{\partial \theta} \right) = \dot{w}_i + \frac{1}{\text{Re}_{0\infty}} \frac{1}{r} \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( \frac{r^2 \mu}{\text{Sc}} \frac{\partial Y_i}{\partial r} \right) + \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left( \frac{\mu \sin \theta}{\text{Sc} r} \frac{\partial Y_i}{\partial \theta} \right) \right] \quad \cdot \quad i = 1, \dots, \text{NS} \quad (13)$$

Transverse Momentum ( $\theta$  Direction):

$$\rho \left( v \frac{\partial u}{\partial r} + \frac{u}{r} \frac{\partial u}{\partial \theta} + \frac{uv}{r} \right) = - \frac{1}{r} \frac{\partial P}{\partial \theta} + \frac{1}{\text{Re}_{0\infty}} \left\{ \frac{\partial}{\partial r} \left[ \mu r \frac{\partial}{\partial r} \left( \frac{u}{r} \right) + \frac{\mu}{r} \frac{\partial v}{\partial \theta} \right] + \frac{1}{r^2} \frac{\partial}{\partial \theta} \left[ \frac{4}{3} \mu \left( \frac{\partial u}{\partial \theta} + v \right) - \frac{2}{3} \mu \left( r \frac{\partial v}{\partial r} + v + u \cot \theta \right) \right] + \frac{3\mu}{r} \left[ r \frac{\partial}{\partial r} \left( \frac{u}{r} \right) + \frac{1}{r} \frac{\partial v}{\partial \theta} \right] + \frac{2\mu \cot \theta}{r^2} \left[ \frac{\partial u}{\partial \theta} - u \cot \theta \right] \right\} \quad \cdot \quad (14)$$

Radial Momentum ( $r$  Direction):

$$\rho \left( v \frac{\partial v}{\partial r} + \frac{u}{r} \frac{\partial v}{\partial \theta} - \frac{u^2}{r} \right) = - \frac{\partial P}{\partial r} + \frac{1}{\text{Re}_{0\infty}} \left\{ \frac{\partial}{\partial r} \left[ \frac{4}{3} \mu \frac{\partial v}{\partial r} - \frac{2}{3} \frac{\mu}{r} \left( \frac{\partial u}{\partial \theta} + 2v + u \cot \theta \right) \right] + \frac{1}{r} \frac{\partial}{\partial \theta} \left[ \mu r \frac{\partial}{\partial r} \left( \frac{u}{r} \right) + \frac{\mu}{r} \frac{\partial v}{\partial \theta} \right] + \frac{4\mu}{r} \left( \frac{\partial v}{\partial r} - \frac{v}{r} \right) - \frac{\mu}{r^2} \left[ 2 \frac{\partial u}{\partial \theta} + 2u \cot \theta - r^2 \cot \theta \frac{\partial}{\partial r} \left( \frac{u}{r} \right) - \frac{\partial v}{\partial \theta} \cot \theta \right] \right\} \quad \cdot \quad (15)$$

Enthalpy:

$$\begin{aligned}
\rho \left( v \frac{\partial h}{\partial r} + \frac{u}{r} \frac{\partial h}{\partial \theta} \right) = & v \frac{\partial P}{\partial r} + \frac{u}{r} \frac{\partial P}{\partial \theta} + \frac{\mu}{\text{Re}_{0\infty}} \left[ 2 \left\{ \left( \frac{\partial v}{\partial r} \right)^2 + \left( \frac{1}{r} \frac{\partial u}{\partial \theta} + \frac{v}{r} \right)^2 \right. \right. \\
& \left. \left. + \left( \frac{v}{r} + \frac{u \cot \theta}{r} \right)^2 \right\} + \left\{ \frac{1}{r} \frac{\partial v}{\partial \theta} + r \frac{\partial}{\partial r} \left( \frac{u}{r} \right)^2 \right\} - \frac{2}{3} \frac{\mu}{\text{Re}_{0\infty}} \left[ \frac{\partial v}{\partial r} + \frac{1}{r} \frac{\partial u}{\partial \theta} \right. \right. \\
& \left. \left. + \frac{2v}{r} + \frac{u \cot \theta}{r} \right]^2 + \frac{1}{\text{Re}_{0\infty} \text{Pr}} \left\{ \frac{1}{r^2} \frac{\partial}{\partial r} \left( \mu r^2 \frac{\partial h}{\partial r} \right) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} \left( \frac{\mu \sin \theta}{r} \frac{\partial h}{\partial \theta} \right) \right. \right. \\
& \left. \left. + (\text{Le} - 1) \frac{1}{r^2} \frac{\partial}{\partial r} \sum_{i=1}^{\text{NS}} \left( \mu r^2 h_i \frac{\partial Y_i}{\partial r} \right) \right. \right. \\
& \left. \left. + (\text{Le} - 1) \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} \sum_{i=1}^{\text{NS}} \left( \frac{\mu h_i \sin \theta}{r} \frac{\partial Y_i}{\partial \theta} \right) \right\} \right] . \tag{16}
\end{aligned}$$

State:

$$P = \rho T \frac{\bar{R} \bar{T}_{0\infty}}{\bar{V}_{\infty}^2} \sum_{i=1}^{\text{NS}} \frac{Y_i}{W_i} . \tag{17}$$

In the previous equations,  $\text{Le}_{ij}$  is assumed to be the same for all species pairs  $i, j$  and is, therefore, replaced with  $\text{Le}$ . Similarly,  $\text{Sc}_{ij}$  is replaced with  $\text{Sc}$ .

The local similarity approximation for the zone near the axis of symmetry is given by equations (18) through (24). The validity of this approximation was demonstrated by Kao [7].

$$u(r, \theta) = u_1(r) \sin \theta \quad (18)$$

$$v(r, \theta) = v_1(r) \cos \theta \quad (19)$$

$$h(r, \theta) = h_1(r) \quad (20)$$

$$\rho(r, \theta) = \rho_1(r) \quad (21)$$

$$P(r, \theta) = P_1(r) + P_2(r) \sin^2 \theta \quad (22)$$

$$\mu(r, \theta) = \mu_1(r) \quad (23)$$

$$Y_i(r, \theta) = Y_{1i}(r) \quad . \quad (24)$$

Equations (18) through (24) are used to reduce the governing equations to a set of nonlinear, coupled, ordinary differential equations which can be solved rapidly.

A transformation is now made in the normal coordinate by defining

$$\eta \equiv \frac{r - 1}{r_\infty - 1} = \frac{n}{n_\infty} \quad , \quad (25)$$

where  $r_\infty$  is the nondimensional distance from the origin to the freestream and  $n_\infty$  is the nondimensional distance from the body to the freestream (Fig. 1b). The values of  $r_\infty$  and  $n_\infty$  are unknown a priori. They are determined as part of the solution. This transformation, equation (25), keeps the body at  $\eta = 0$  and the freestream at  $\eta = 1$ .

By substituting equations (18) through (25) into the governing equations and equating the coefficients of like functions of  $\theta$ , one obtains the following system of ordinary differential equations:

Cylindrical Coordinates:

$$\frac{\rho'_1}{\rho_1} = -\frac{v'_1}{v_1} - \frac{n_\infty}{1 + \eta n_\infty} \left( 1 + \frac{u_1}{v_1} \right) \quad . \quad (26)$$

$$\begin{aligned} \frac{u''_1}{n_\infty^2} &= \frac{\text{Re}_{0\infty} \rho_1}{\mu_1} \left( \frac{v_1 u'_1}{n_\infty} + \frac{u_1^2 + u_1 v_1}{1 + \eta n_\infty} \right) + \frac{2 P_2 \text{Re}_{0\infty}}{\mu_1 (1 + \eta n_\infty)} \\ &+ \frac{u_1 + v_1}{(1 + \eta n_\infty)} \left[ \frac{\mu'_1}{\mu_1 n_\infty} + \frac{7}{3(1 + \eta n_\infty)} \right] - \frac{u'_1}{n_\infty} \left( \frac{2}{1 + \eta n_\infty} + \frac{\mu'_1}{\mu_1 n_\infty} \right) \\ &+ \frac{v'_1}{3n_\infty (1 + \eta n_\infty)} \quad . \end{aligned} \quad (27)$$

$$\begin{aligned} \frac{v''_1}{n_\infty^2} &= \frac{3}{4} \frac{\text{Re}_{0\infty}}{n_\infty \mu_1} (P'_1 + \rho_1 v_1 v'_1) - \frac{v'_1}{n_\infty} \left( \frac{\mu'_1}{\mu_1 n_\infty} + \frac{1}{1 + \eta n_\infty} \right) - \frac{u'_1}{4n_\infty (1 + \eta n_\infty)} \\ &+ \frac{(u_1 + v_1)}{1 + \eta n_\infty} \left[ \frac{\mu'_1}{2\mu_1 n_\infty} + \frac{7}{4(1 + \eta n_\infty)} \right] \quad . \end{aligned} \quad (28)$$

$$\frac{P'_2}{n_\infty} = -\frac{P'_1}{n_\infty} + \frac{\rho_1 u_1}{1 + \eta n_\infty} (u_1 + v_1) \quad . \quad (29)$$

$$\begin{aligned}
\frac{\rho_1 v_1 h'_1}{n_\infty} (1 + \eta n_\infty)^2 &= (1 + \eta n_\infty)^2 \left[ \frac{v_1 P'_1}{n_\infty} + \frac{2\mu_1}{\text{Re}_{0\infty}} \frac{v_1'^2}{n_\infty^2} \right] + \frac{2\mu_1}{\text{Re}_{0\infty}} (u_1 + v_1)^2 \\
&\quad - \frac{2}{3} \frac{\mu_1}{\text{Re}_{0\infty}} \left[ \frac{(1 + \eta n_\infty)}{n_\infty} v'_1 + (u_1 + v_1) \right]^2 \\
&\quad + \frac{(1 + \eta n_\infty)}{\text{Re}_{0\infty} \text{Pr } n_\infty} \left\{ \mu_1 h'_1 + \frac{(1 + \eta n_\infty)}{n_\infty} (h'_1 \mu'_1 + \mu_1 h'_1) \right. \\
&\quad + (\text{Le} - 1) \left[ \left( \mu_1 + \frac{\mu'_1 (1 + \eta n_\infty)}{n_\infty} \right) \sum_{i=1}^{\text{NS}} h_i Y'_{1i} \right. \\
&\quad \left. \left. + \frac{\mu_1 (1 + \eta n_\infty)}{n_\infty} \sum_{i=1}^{\text{NS}} (h'_i Y'_{1i} + h_i Y''_{1i}) \right] \right\} . \tag{30}
\end{aligned}$$

$$\begin{aligned}
\frac{\rho_1 v_1 Y'_{1i}}{n_\infty} &= \dot{w}_i + \frac{1}{(1 + \eta n_\infty) \text{Re}_{0\infty} \text{Sc}} \left[ \frac{\mu_1 Y'_{1i}}{n_\infty} + \frac{\mu'_1 Y'_{1i} (1 + \eta n_\infty)}{n_\infty^2} \right. \\
&\quad \left. + \frac{\mu_1 (1 + \eta n_\infty) Y''_{1i}}{n_\infty^2} \right] \quad i = 1, \dots, \text{NS} . \tag{31}
\end{aligned}$$

$$P_1 = \rho_1 T_1 \left( \frac{\bar{R} \bar{T}_{0\infty}}{\bar{V}_\infty^2} \right) \sum_{i=1}^{\text{NS}} \left( \frac{Y_{1i}}{W_i} \right) . \tag{32}$$

Here, a prime denotes differentiation with respect to  $\eta$ .

Spherical Coordinates:

$$\frac{\rho'_1}{\rho_1} = -\frac{v'_1}{v_1} - \frac{2n_\infty}{1 + \eta n_\infty} \left( 1 + \frac{u_1}{v_1} \right) \quad (33)$$

$$\begin{aligned} \frac{u''_1}{n_\infty^2} = & \frac{\text{Re}_{0\infty} \rho_1}{\mu_1} \left( \frac{v_1 u'_1}{n_\infty} + \frac{u_1^2 + u_1 v_1}{1 + \eta n_\infty} \right) + \frac{2P_2 \text{Re}_{0\infty}}{\mu_1 (1 + \eta n_\infty)} \\ & + \frac{u_1 + v_1}{(1 + \eta n_\infty)} \left[ \frac{\mu'_1}{\mu_1 n_\infty} + \frac{8}{3(1 + \eta n_\infty)} \right] - \frac{u'_1}{n_\infty} \left( \frac{2}{1 + \eta n_\infty} + \frac{\mu'_1}{\mu_1 n_\infty} \right) \\ & + \frac{v'_1}{3n_\infty (1 + \eta n_\infty)} \quad . \end{aligned} \quad (34)$$

$$\begin{aligned} \frac{v''_1}{n_\infty^2} = & \frac{3}{4} \frac{\text{Re}_{0\infty}}{n_\infty \mu_1} (P'_1 + \rho_1 v_1 v'_1) - \frac{v'_1}{n_\infty} \left( \frac{\mu'_1}{\mu_1 n_\infty} + \frac{2}{1 + \eta n_\infty} \right) - \frac{u'_1}{2n_\infty (1 + \eta n_\infty)} \\ & + \frac{(u_1 + v_1)}{(1 + \eta n_\infty)} \left[ \frac{\mu'_1}{\mu_1 n_\infty} + \frac{7}{2(1 + \eta n_\infty)} \right] \quad . \end{aligned} \quad (35)$$

$$\frac{P'_2}{n_\infty} = -\frac{P'_1}{n_\infty} + \frac{\rho_1 u_1}{1 + \eta n_\infty} (u_1 + v_1) \quad . \quad (36)$$



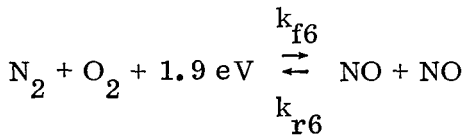
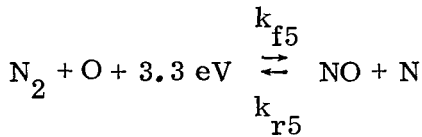
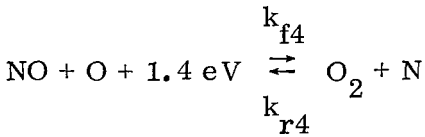
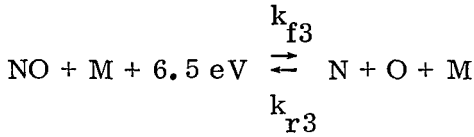
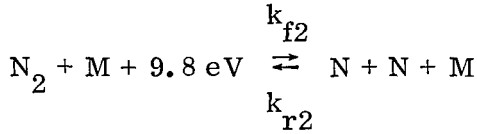
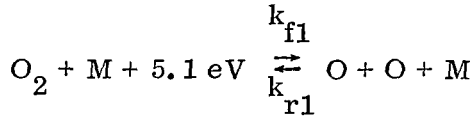
$$\begin{aligned}
\frac{\rho_1 v_1 h_1'}{n_\infty} (1 + \eta n_\infty)^2 &= (1 + \eta n_\infty)^2 \left[ \frac{v_1 P_1'}{n_\infty} + \frac{2\mu_1}{\text{Re}_{0\infty}} \frac{v_1'^2}{n_\infty^2} \right] + \frac{4\mu_1}{\text{Re}_{0\infty}} (u_1 + v_1)^2 \\
&- \frac{2}{3} \frac{\mu_1}{\text{Re}_{0\infty}} \left[ \frac{(1 + \eta n_\infty)}{n_\infty} v_1' + 2(u_1 + v_1) \right]^2 \\
&+ \frac{(1 + \eta n_\infty)}{\text{Re}_{0\infty} \text{Pr } n_\infty} \left\{ 2\mu_1 h_1' + \frac{(1 + \eta n_\infty)}{n_\infty} (\mu_1' h_1' + \mu_1' h_1'') \right. \\
&+ (\text{Le} - 1) \left[ \left( 2\mu_1 + \frac{\mu_1' (1 + \eta n_\infty)}{n_\infty} \right) \sum_{i=1}^{\text{NS}} h_i Y_{1i}' \right. \\
&\left. \left. + \frac{\mu_1 (1 + \eta n_\infty)}{n_\infty^2} \sum_{i=1}^{\text{NS}} (h_i' Y_{1i}' + h_i Y_{1i}'') \right] \right\} . \tag{37}
\end{aligned}$$

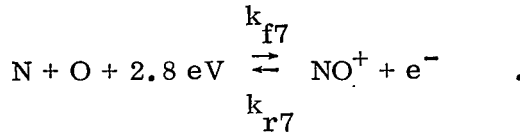
$$\begin{aligned}
\frac{\rho_1 v_1 Y_{1i}'}{n_\infty} &= \dot{w}_i + \frac{1}{(1 + \eta n_\infty) \text{Re}_{0\infty} \text{Sc}} \left[ \frac{2\mu_1 Y_{1i}'}{n_\infty} + \frac{\mu_1 Y_{1i}' (1 + \eta n_\infty)}{n_\infty^2} \right. \\
&\left. + \frac{\mu_1 (1 + \eta n_\infty) Y_{1i}''}{n_\infty^2} \right] \quad i = 1, \dots, \text{NS} . \tag{38}
\end{aligned}$$

$$P_1 = \rho_1 T_1 \frac{\bar{R} \bar{T}_{0\infty}}{\bar{V}_\infty^2} \sum_{i=1}^{\text{NS}} \left( \frac{Y_{1i}}{W_i} \right) . \tag{39}$$

Equations (26) through (32) and (33) through (39) constitute two sets of non-linear ordinary differential equations for cylindrical and spherical stagnation regions, respectively. Each of these sets contain  $6 + NS$  equations and  $8 + 2NS$  unknowns ( $\mu_1$ ,  $v_1$ ,  $P_1$ ,  $P_2$ ,  $\rho_1$ ,  $T_1$ ,  $h_1$ ,  $\mu_1$ ,  $\dot{w}_i$ , and  $Y_{1i}$ ). The required additional information and equations are given below for the mass production rates,  $\dot{w}_i$ , enthalpy,  $h_1$ , and viscosity,  $\mu_1$ , of a dissociated and ionized air mixture. These equations, like the equation of state above, are independent of the coordinate system.

c. Air Chemistry. The freestream air is assumed to consist of  $N_2$  and  $O_2$  molecules only. The air model used for the shock layer consists of seven species and the seven chemical reactions as follows [5]:





These reactions are written so that the forward reactions are endothermic; the net amounts of energy required to produce the reactions are given on the left side of the equations. The first three reactions are the neutral-particle dissociation-recombination reactions in which the energy of dissociation in the forward reaction is taken primarily from kinetic energy by means of a collision with a "catalytic" molecule M; the chemical energy released in the recombination (reverse reaction) is converted primarily to kinetic energy in a three-body collision involving a catalytic molecule M. The M molecule can be any of the six molecular or atomic species present in the air mixture. The quantities  $k_{fj}$  and  $k_{rj}$  are the temperature-dependent forward and reverse reaction rates, respectively, for the jth reaction. The experimentally determined reaction rates recommended by Wray [5] are used, together with the concentrations of all constituents, to obtain the mass production rates,  $\dot{w}_i$ , of each constituent [9]. The reader is referred to Reference 10 for details.

d. Enthalpy. The specific enthalpy of a mixture of gases is given in dimensional form as follows:

$$\bar{h} = \bar{P} \bar{v} + \sum_{i=1}^{NS} Y_i \bar{e}_i , \quad (40)$$

where

$$\bar{e}_i = \bar{e}_i^0 + \bar{e}_{T_i} + \bar{e}_{R_i} + \bar{e}_{v_i} \quad (41)$$

and

$\bar{e}_i^0$  = specific energy of formation of species i at the reference temperature (zero absolute)

$\bar{e}_{T_i}$  = specific energy of random translation

$\bar{e}_{R_i}$  = specific energy of rotation (for diatomic molecule)

$\bar{e}_{v_i}$  = specific energy of vibration (for diatomic molecules).

We assume that the energy of electronic excited states is negligible, that the rotational state is fully excited, but that the vibrational state is partially excited. Values for these quantities are given by Vincenti and Kruger [9]:

$$\bar{e}_{T_i} = \frac{3}{2} \bar{R}_i \bar{T} \quad (42)$$

$$\bar{e}_{R_i} = \bar{R}_i \bar{T} \quad (43)$$

$$\bar{e}_{v_i} = \frac{\bar{R}_i \Theta_{v_i}}{e^{\Theta_{v_i}/\bar{T}} - 1}, \quad (44)$$

where

$\Theta_{v_i}$  = characteristic temperature for vibration of diatomic species i

$$\Theta_{v_{O_2}} = 2270 \text{ K}$$

$$\Theta_{v_{N_2}} = 3390 \text{ K}$$

$$\Theta_{v_{NO}} = 2740 \text{ K}$$

$$\Theta_{v_{NO^+}} = 2740 \text{ K} \quad .$$

Equation (40) now becomes

$$\bar{h} = \frac{5}{2} \bar{R} \bar{T} \sum_{i=1}^{NS} \frac{Y_i}{W_i} + \bar{R} \sum_i \frac{Y_i}{W_i} \left( \bar{T} + \frac{\bar{\Theta}_{v_i}}{e^{\bar{\Theta}_{v_i}/\bar{T}} - 1} \right) + \sum_{i=1}^{NS} \bar{e}_i^0 \quad . \quad (45)$$

(for  
diatomic  
molecules)

Nondimensionalizing equation (45), we obtain

$$h = \frac{5}{2} \frac{\bar{R} \bar{T} \bar{T}_{0\infty}}{\bar{V}_\infty^2} \sum_{i=1}^{NS} \frac{Y_i}{W_i} + \frac{\bar{R} \bar{T}_{0\infty}}{\bar{V}_\infty^2} \sum_i \frac{Y_i}{W_i} \left( T + \frac{\Theta_{v_i}}{e^{\Theta_{v_i}/T} - 1} \right) + \frac{1}{\bar{V}_\infty^2} \sum_{i=1}^{NS} Y_i \bar{e}_i^0 \quad . \quad (46)$$

(for  
diatomic  
molecules)

e. Viscosity. The Sutherland formula for the viscosity of air gives acceptable results at moderate temperatures, but it fails at the extremely high temperatures encountered in hypersonic flight. The viscosity begins deviating from the Sutherland value due to the onset of dissociation at approximately 3000 K. The viscosity near the wall is also affected by the extremely low pressures encountered at high altitude [11]. This effect is due to velocity slip at the wall (see next section). The Sutherland formula, equation (47), is recommended for temperatures less than 3000 K.

$$\bar{\mu}_{su} = \frac{1.458 \times 10^{-5} \bar{T}^{3/2}}{110.4 + \bar{T}} \quad [\text{gm/cm sec}] \quad . \quad (47)$$

This formula is also used in the computer program (Appendix A) to calculate  $\bar{\mu}_{0\infty}$  using  $\bar{T}_{0\infty}$  computed from the adiabatic relation for temperature,

although it is realized that neither  $\bar{\mu}_{su}$  nor  $\bar{T}_{0\infty}$  has valid physical meaning at extremely high stagnation enthalpy. This use is justified because  $\bar{\mu}_{0\infty}$  is used only for nondimensionalizing  $\bar{\mu}$  and for computing  $Re_{0\infty}$ .

The viscosity of dissociated and ionized air is approximated in Reference 6 using a simple summation formula for a mixture of hard spherical molecules:

$$\frac{\bar{\mu}}{\bar{\mu}_{su}} = \sum_{i=1}^{NS} Y_i \sqrt{\frac{W_R}{W_i}} \frac{\bar{\lambda}_i}{\bar{\lambda}_R}, \quad (48)$$

where

$\bar{\mu}_{su}$  = viscosity at same temperature from Sutherland formula

$W_R$  = equivalent molecular weight of undissociated air

$\bar{\lambda}_i$  = mean free path of species  $i$

$\bar{\lambda}_R$  = mean free path of undissociated air molecules.

The ratio of mean free paths in equation (48) is given by

$$\frac{\bar{\lambda}_R}{\bar{\lambda}_i} = \sum_{j=1}^{NS} \left[ Y_j \frac{W_R}{W_j} \frac{\bar{S}_{ij}}{\bar{S}_R} \left( \frac{1 + \frac{W_i}{W_j}}{2} \right)^{1/2} \right], \quad (49)$$

where

$\bar{S}_{ij}$  = collision cross section for particle  $i$  with particle  $j$

$\bar{S}_R$  = collision cross section for undissociated air molecules.

The collision cross sections are tabulated as a function of temperature in Reference 6.

The viscosity given by equation (48) is a function of the composition of the gas mixture and also of the temperature via the dependence of  $\bar{S}_R$  and  $\bar{S}_{ij}$  on temperature. Since in the present analysis the air is not in chemical equilibrium, the viscosity obtained from the computer program at a given temperature differs greatly from that shown in Reference 6 for equilibrium conditions. As the mass fractions of the components of air approach their undissociated values, the numerical value of the viscosity ratio in equation (48) approaches 1.0, i.e.,  $\bar{\mu}$  approaches the Sutherland value as expected.

3. Boundary Conditions. To solve the governing equations given in the previous section, freestream and wall boundary conditions are required.

Freestream ( $\eta = 1$ ):

The air at the freestream boundary is in its undisturbed state.

$$u_1 = 1 \quad ,$$

$$v_1 = -1 \quad ,$$

$$\rho_1 = 1 \quad ,$$

$$T_1 = T_\infty \quad ,$$

$$P_1 = P_\infty \quad ,$$

$$P_2 = 0 \quad ,$$

$$Y_{1N_2} = 0.767 \quad ,$$

$$Y_{1O_2} = 0.233 \quad ,$$

$$Y_{1NO} = Y_{1N} = Y_{1O} = Y_{1NO^+} = Y_{1e^-} = 0 \quad .$$

Wall ( $\eta = 0$ ):

In high-altitude, low-Reynolds number flight, the continuum model of the gas breaks down in regions of large gradients of the physical properties near the wall. Hence, the Navier-Stokes description is invalid for the gas layer near the wall (Knudsen layer) with thickness on the order of the mean free path [11]. Also, the familiar continuum zero-velocity and zero-temperature-jump wall boundary conditions are not applicable. Although the Navier-Stokes equations are invalid near the wall, they can still be used, down to quite low Reynolds numbers, to describe the outer flow field if the proper boundary conditions are used at the outer edge of the Knudsen layer. These boundary conditions, known as slip conditions, are the mean velocity, temperature, and species mass fractions. To compute these slip conditions, a kinetic theory approach must be used for the Knudsen layer. The boundary conditions for the Knudsen layer are the mean slip conditions at the outer edge and the kinetic gas-surface conditions at the wall.

The Boltzmann equation is the governing equation for the kinetic theory description of a flow field. However, due to the difficulty in solving the Boltzmann equation for the Knudsen layer, we resorted to an approximate kinetic theory slip model. Reference 4 gives the details of the derivation of the slip conditions for a multicomponent reacting gas. By matching the species, momenta, and energy fluxes at the outer edge of the Knudsen layer to the difference between the incident and reflected fluxes at the wall, the jump in the desired properties across the Knudsen layer is obtained. The fluxes are calculated by taking moments of the velocity distribution function which is approximated by using a Chapman-Enskog expansion for a multicomponent mixture. The species flux is greatly affected by the catalytic nature of the wall. The wall is assumed to be catalytic with respect to recombination of dissociated molecules. Equations are obtained for a partially catalytic wall. The extremes of noncatalytic and fully catalytic walls are easily obtained from the equations for a partially catalytic wall. The resulting nondimensional equations for slip velocity, temperature, and species are as follows:

$$\begin{aligned}
 u_s = & \frac{\mu_s}{P_w \text{Re}_\infty M_\infty} \left[ \sqrt{\pi} \left( \frac{2-\sigma}{2\sigma} \right) \sqrt{\frac{2T_s}{\gamma T_\infty}} \left( \frac{\partial u}{\partial r} + \frac{1}{r} \frac{\partial v}{\partial \theta} \right)_s \right. \\
 & + \frac{1}{5(\gamma-1) M_\infty \text{Pr} T_\infty} \left( \frac{1}{r} \frac{\partial T}{\partial \theta} \right)_s \sum_i \sqrt{\frac{W_i}{W}} \\
 & \left. - \frac{P_w M_\infty \text{Le}}{\rho_w \text{Pr}} \sum_j \left( \frac{1}{r} \frac{\partial Y_j}{\partial \theta} \right)_s \sum_i \sqrt{\frac{W_i}{W}} \right] . \quad (50)
 \end{aligned}$$



$$\begin{aligned}
T_s = & \left( T_w \sum_i \left\{ \frac{\rho Y_i}{\left[ W_i \sum_j \frac{Y_j}{W_j} \right]^{3/2}} \left[ 1 + \frac{\mu}{3P_i Re_{0\infty}} \left( \frac{1}{r} \frac{\partial u}{\partial \theta} - 2 \frac{\partial v}{\partial r} \right) \right. \right. \right. \\
& + \left. \frac{\sqrt{\pi}}{Y_i} \left( \frac{2-\sigma}{\sigma} \right) \frac{M_\infty}{Re_{0\infty}} \frac{Le}{Pr} \left[ W_i \sum_j \frac{Y_j}{W_j} \right] \frac{\mu}{\rho} \sqrt{\frac{\gamma W_i T_\infty}{2WT}} \sum_j \frac{\partial Y_j}{\partial r} \left. \right] \right\}_s \\
& + \left\{ \frac{2-\sigma}{\sigma} \sqrt{\pi} \left[ \frac{M_\infty}{Pr Re_{0\infty}} \frac{\gamma}{\gamma-1} \frac{\mu}{2} \sqrt{\frac{\gamma T_\infty}{2T}} \frac{\partial T}{\partial r} \right. \right. \\
& - \left. \left. \frac{5P}{2\rho} \frac{\mu}{Re_{0\infty}} M_\infty^3 \gamma T_\infty \sqrt{\frac{\gamma T_\infty}{2T}} \frac{Le}{Pr} \sum_j \frac{\partial Y_j}{\partial r} \right] \right\}_s \Bigg) / \\
& \left\{ \sum_i \frac{\rho Y_i}{\left[ W_i \sum_j \left( \frac{Y_j}{W_j} \right) \right]^{3/2}} \left[ 1 + \frac{\mu}{2P_i Re_{0\infty}} \left( \frac{1}{r} \frac{\partial u}{\partial \theta} - 2 \frac{\partial v}{\partial r} \right) \right] \right\}_s \quad (51)
\end{aligned}$$

$$\begin{aligned}
Y_i^s = & \frac{\frac{2-\gamma_i}{2\gamma_i} \frac{M_\infty}{Re_{0\infty}} \frac{Le}{Pr} \frac{\mu_s}{\rho_s} \sqrt{\frac{2\pi\gamma W_i T_\infty}{WT_s}} \sum_j \left( \frac{\partial Y_j}{\partial r} \right)_s}{1 + \frac{M_\infty^2}{Re_{0\infty}} \left( \frac{\mu}{3\rho T Y_i} \right)_s \frac{\gamma W_i T_\infty}{W} \left( \frac{1}{r} \frac{\partial u}{\partial \theta} - 2 \frac{\partial v}{\partial r} \right)_s} \quad (52)
\end{aligned}$$

The superscript, s, on  $Y_i^s$  denotes the value of  $Y_i$  at the outer edge of the Knudsen layer.

The similarity equations (18) through (24) and the coordinate transformation equation (25) are used in equations (50) through (52) to yield the following slip equations:

$$u_{1s} = \frac{\mu_{1s}}{P_{1w} Re_{0\infty} M_{\infty}} \left[ \sqrt{\pi} \left( \frac{2-\sigma}{2\sigma} \right) \sqrt{\frac{2T_1}{\gamma T_{\infty}}} \left( \frac{u'_1}{\eta_{\infty}} \right) \right]_s, \quad (53)$$

$$\begin{aligned} T_{1s} = & \left( T_w \sum_i \left\{ \frac{\rho_1 Y_{1i}}{\left[ W_i \sum_j \frac{Y_{1j}}{W_j} \right]^{3/2}} \left[ 1 + \frac{\mu_1}{3P_{1i} Re_{0\infty}} \left( \frac{u_1}{1 + \eta n_{\infty}} - \frac{2v'_1}{n_{\infty}} \right) \right. \right. \right. \\ & + \frac{\sqrt{\pi}}{Y_{1i}} \left( \frac{2-\sigma}{\sigma} \right) \frac{M_{\infty}}{Re_{0\infty}} \frac{Le}{Pr} \left[ W_i \sum_j \frac{Y_{1j}}{W_j} \right] \frac{\mu_1}{\rho_1} \sqrt{\frac{\gamma W_i T_{\infty}}{2WT_1}} \sum_j \frac{Y'_{1j}}{n_{\infty}} \left. \right] \Bigg\} \\ & + \frac{2-\sigma}{\sigma} \sqrt{\pi} \left[ \frac{M_{\infty}}{Pr Re_{0\infty}} \frac{\gamma}{\gamma-1} \frac{\mu_1}{2} \sqrt{\frac{\gamma T_{\infty}}{2T_1}} \times \frac{T'_1}{n_{\infty}} \right. \\ & \left. \left. - \frac{5P_1}{2\rho_1} \frac{\mu_1}{Re_{0\infty}} M_{\infty}^3 \gamma T_{\infty} \sqrt{\frac{\gamma T_{\infty}}{2T_1}} \frac{Le}{Pr} \sum_j \frac{Y'_{1j}}{n_{\infty}} \right] \right) \Bigg\} \\ & \div \left\{ \sum_i \frac{\rho_1 Y_{1i}}{\left[ W_i \sum_j \frac{Y_{1j}}{W_j} \right]^{3/2}} \left[ 1 + \frac{\mu_1}{2P_{1i} Re_{0\infty}} \left( \frac{u_1}{1 + \eta n_{\infty}} - \frac{2v'_1}{n_{\infty}} \right) \right] \right\}_s, \end{aligned} \quad (54)$$

and

$$Y_{1i}^s = \frac{\frac{2 - \gamma_i}{2\gamma_i} \frac{M_\infty}{Re_{0\infty}} \frac{Le}{Pr} \frac{\mu_{1s}}{\rho_{1s}} \sqrt{\frac{2\pi\gamma_i W_i T_\infty}{W T_{1s}}} \sum_j \left( \frac{Y'_{1j}}{n_\infty} \right)_s}{1 + \frac{M_\infty^2}{Re_{0\infty}} \left( \frac{\mu_1}{3\rho_1 T_1 Y_{1i}} \right)_s \frac{\gamma_i W_i T_\infty}{W} \left( \frac{u_1}{1 + \eta n_\infty} - \frac{2v'_1}{n_\infty} \right)_s} \quad (55)$$

The quantity  $\gamma_i$  in equation (55) is the recombination coefficient for the  $i$  species; it is a measure of the catalyticity of the wall. The value of  $\gamma_i$  varies from 0 (noncatalytic wall) to 1 (fully catalytic wall).

The computer program used in this report has the fully catalytic wall and noncatalytic wall options, but it does not, in the present form, have a partially catalytic wall option [for a given catalyticity  $\gamma_i$ , equation (55) would give  $Y_i^s$ ]. In the fully catalytic wall option, the wall is assumed to be catalytic only with respect to recombination of neutral atomic species; it is assumed to be noncatalytic with respect to recombination of the charged particles  $NO^+$  and  $e^-$ . The wall boundary conditions on  $Y_i$  for these special cases are given as follows:

Noncatalytic Wall:

For a noncatalytic surface ( $\gamma_i = 0$ ), equation (55) reduces to

$$\sum_j (Y'_{1j})_s = 0 \quad (56)$$

A sufficient condition for equation (56) to be satisfied is that

$$(Y'_{ij})_s = 0 \quad j = 1, \dots, NS \quad (57)$$

Equation (57) is also the most physically plausible means by which equation (56) can be satisfied; therefore, equation (57) is taken as the noncatalytic boundary condition for species mass fractions.

### Fully Catalytic Wall:

For the fully catalytic wall, the surface is assumed to be fully catalytic with respect to recombination of neutral atomic species, but it is assumed to be noncatalytic with respect to recombination of the charged particles  $\text{NO}^+$  and  $e^-$ . Therefore, the effect of the wall is to drive the gas towards its freestream composition, except for the charged particles  $\text{NO}^+$  and  $e^-$ . The following boundary conditions are then obtained:

$$\left(Y_{\text{N}_2}\right)_s = 0.767 \quad (58)$$

$$\left(Y_{\text{O}_2}\right)_s = 0.233 \quad (59)$$

$$\left(Y_{\text{NO}}\right)_s = \left(Y_{\text{N}}\right)_s = \left(Y_{\text{O}}\right)_s = 0 \quad (60)$$

$$\left(Y'_{\text{NO}^+}\right)_s = \left(Y'_{e^-}\right)_s = 0 \quad . \quad (61)$$

Equations (58), (59), and (61) produce the impossible result that

$$\sum_{i=1}^{\text{NS}} Y_i > 1 \quad ;$$

however, because  $\left(Y_{\text{NO}^+}\right)_s$  and  $\left(Y_{e^-}\right)_s$  are usually very small, this error is tolerable.

## B. Method of Solution

Equations (26) through (32) and (33) through (39) together with equations (46) and (48) constitute two sets of nonlinear, coupled ordinary differential equations with boundary conditions previously given. The first order equations are solved by direct numerical quadrature, and the second order equations are

integrated by a finite difference method known as the successive accelerated replacement method which is an iterative scheme that starts from a guessed solution. The salient feature of this method, proposed by Lieberstein [13], is that the successive corrections applied to the flow profiles in each iteration are controlled by acceleration factors which are used to increase the rate of convergence of the computed quantities. Thus, this method can be successfully applied even if the initial, guessed profiles do not approximate the converged solutions very well. For the present application, this statement holds true in the midrange of the flow regime for which our analysis is applicable. However, divergence problems are encountered at the continuum end of the regime (Appendix A).

### C. Computer Program

Details of the computer program are found in Reference 10, while Appendix A is a current users manual.

## III. RESULTS

Results from the computer program, with the modifications introduced in this report, are first compared with the "old" program. Then, the program output is compared with some available experimental data. Finally, some general results from the program and input data for running the program are presented.

### A. Effect of Computer Program Modifications

The program described as the old program includes all the analysis in this report except the constitutive equations (46) and (48) for enthalpy and viscosity, respectively. The old enthalpy equation assumed a fully excited vibrational state for diatomic molecules. This assumption is always violated near the freestream and near the wall because temperatures in these regions are less than  $\Theta_v$ . The old viscosity equation (Sutherland) is restricted to non-dissociated air or to temperatures less than approximately 3000 K at equilibrium conditions. This temperature is greatly exceeded in the central part of the shock layer at reentry speeds.

## B. Comparison of Data Between Old Program and Modified Program

Computed values of shock layer thickness,  $n_\infty$ ; wall heat transfer coefficient at the stagnation point,  $C_H$ ; and maximum ratio of viscosity to Sutherland viscosity on the stagnation streamline,  $\mu/\mu_{su}$ , are given in Table 1 for a 30.5-cm radius sphere using (1) the "old" program, (2) the program with enthalpy modification only, and (3) the program with enthalpy and viscosity modifications. The conditions for which the runs were made are listed in Table 1. The thermodynamic properties associated with altitude are obtained throughout this report from Reference 14.

TABLE 1. DATA COMPARISON

$$\text{alt} = 86 \text{ km}$$

$$\bar{r}_b = 30.5 \text{ cm}$$

$$\bar{V}_\infty = 793\,000 \text{ cm/sec}$$

Program Identification	Shock Layer Thickness $n_\infty$	Wall Heat Transfer Coefficient $C_H$	$\left(\frac{\mu}{\mu_{sv}}\right)_{\max}$
1. Old Program	0.1361	0.147	1.0
2. Modified Program (a) New enthalpy computation, equation (46) (b) Sutherland viscosity	0.1409	0.154	1.0
3. Modified Program (a) New enthalpy computation, equation (46) (b) New viscosity, equation (48)	0.1411	0.172	1.37

For these runs, conditions were chosen at which the modifications in viscosity and enthalpy computation would have their greatest effect. These conditions are the ones which yield a high degree of dissociation and ionization, equations (46) and (48), i.e. high speed and high freestream density. Therefore, an altitude was chosen (86 km) where freestream density is near the maximum for which the program will run (see General Results section). The table shows that the enthalpy and viscosity modifications had a significant effect on heat transfer rate and shock layer thickness. The heat transfer rate was increased 17 percent due to the combined effect of both modifications.

### C. Comparison with Experimental Data

Very little experimental data exist for the low density, hypersonic, high stagnation enthalpy flow regime for which this computer program was designed. Therefore, we do not have the detailed comparison with experimental data which is desired to validate the present analysis. However, a limited amount of heat transfer and pressure data for spheres are available from an arc-jet facility at moderate stagnation enthalpy [15], and a larger body of stagnation heat transfer data exists for relatively low enthalpy, and hypersonic flow [16].

Figure 2 compares computed stagnation point heat transfer coefficients,  $C_H$ , for a sphere with experimental data from References 15 and 16. The data in this figure represent several levels of stagnation enthalpy and, hence, several levels of real gas effects, so good agreement among all the data is not expected. The purpose here is to compare the computed results directly with each set of experimental data and to observe the trend in  $C_H$  with changing stagnation enthalpy. A best fit curve through the relatively low enthalpy data of Vidal and Wittliff [16] is shown by the solid line, and the square symbols give the computed values for the same flow conditions. Although the computer program was designed for real gas, chemically reacting flows, it can be run at the low enthalpy conditions of the Vidal and Wittliff data, in which case the computed dissociation levels are very low. The data are plotted as a function of the similarity parameter,  $K^2$ , used by Vidal and Wittliff, and defined as follows:

$$K^2 = \frac{Re_\infty}{\gamma_\infty M_\infty^2 C^*} \quad , \quad (62)$$

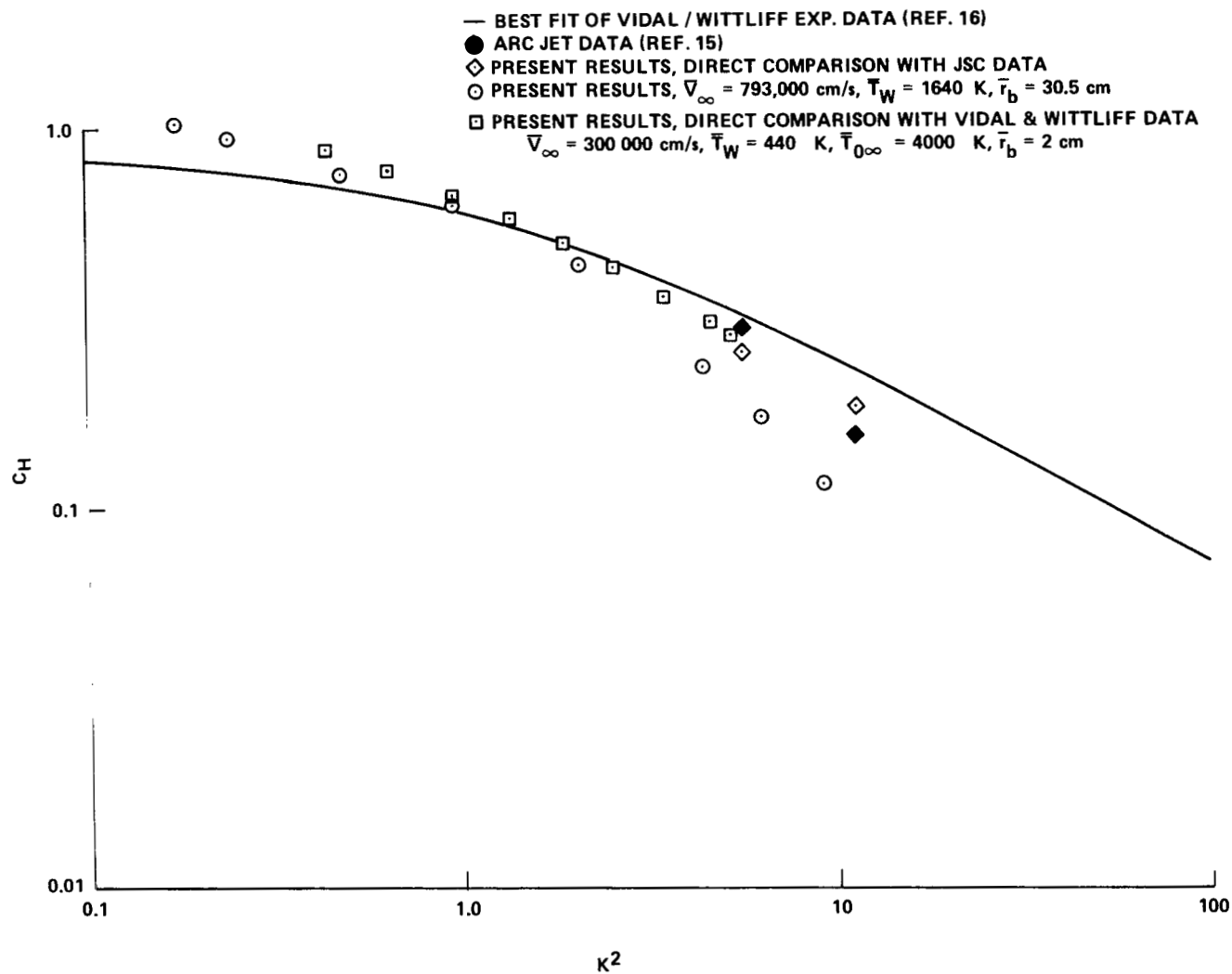


Figure 2. Comparison of present theory with experimental data for stagnation point heating on a sphere.



where

$$C^* = \frac{\bar{\mu}^*}{\bar{\mu}_\infty} \frac{\bar{T}_\infty}{\bar{T}^*} \quad (63)$$

and

$$\bar{T}^* = \frac{1}{2} (\bar{T}_{0\infty} + \bar{T}_w) \quad , \quad (64)$$

where

$\bar{\mu}^*$  = viscosity computed by Sutherland's formula using  $\bar{T}^*$

$\bar{\mu}_\infty$  = viscosity computed by Sutherland's formula using  $\bar{T}_\infty$ .

The computed data do not agree with the mean of the Vidal and Wittliff data as well as we would like, but the computed data points fall within the scatter of the experimental points [16]. The computed values tend to overestimate  $C_H$  at rarefied conditions and underestimate it for denser flow. It is important to note that at the higher enthalpy cases, the wall catalysis can affect the value of  $C_H$  by a factor of 3 or more [4]. Hence, any comparison of experimental and calculated heating must be for similar wall catalysis.

The arc-jet data of Scott [15] are shown by the filled diamond symbols, and computed data for the same conditions are shown by the empty diamond symbols. The flow conditions for these tests are given in Table 2.

The freestream boundary conditions in the computer program were modified to match the dissociated arc-jet freestream. Also, the spheres in the arc-jet tests were coated with teflon to produce a noncatalytic wall and, therefore, the noncatalytic wall option in the computer program was used for comparison with these data points. The noncatalytic wall option was also used for the remainder of the computer data in this figure and the entire report unless otherwise noted.

The comparison of the computed data with Scott's experimental data is considered acceptable.

TABLE 2. ARC-JET FLOW FIELD PROPERTIES

	Case 1	Case 2
Mach number, $M_\infty$	10	8
Reynolds number, $Re_\infty$	380	553
Freestream speed, $\bar{V}_\infty$ (cm/sec)	423 000	412 000
Freestream temperature, $\bar{T}_\infty$ (K)	270	414
Freestream density, $\bar{\rho}_\infty$ (gm/cm <sup>3</sup> )	$3.28 \times 10^{-8}$	$6.41 \times 10^{-8}$
Wall temperature, $\bar{T}_w$ (K)	450	450
Radius of sphere, $\bar{r}_b$ (cm)	5.08	5.08
Species mass fractions in freestream:		
N <sub>2</sub>	0.4138	0.6124
O <sub>2</sub>	$0.6519 \times 10^{-5}$	$0.1084 \times 10^{-4}$
N	0.3514	0.1522
O	0.2348	0.2347
NO	$0.5380 \times 10^{-5}$	$0.3761 \times 10^{-4}$
NO <sup>+</sup>	$0.2221 \times 10^{-5}$	$0.6849 \times 10^{-5}$
e <sup>-</sup>	$0.3850 \times 10^{-10}$	$0.1187 \times 10^{-9}$

Some high-enthalpy computed data, for which the dissociation and ionization levels are very high, are shown in Figure 2 by the circular symbols. These data points fall considerably below the lower enthalpy data at the continuum end of the flow regime (at large values of  $K^2$ ) where there is a consistent trend of decreasing  $C_H$  with increasing enthalpy for a given  $K^2$ . The parameter  $K^2$  does not account for real gas effects and, therefore, should not be expected to correlate data with widely different enthalpy levels.

Acceptable comparisons have also been obtained with other theoretical predictions by Kumar and Jain [3] who used an analysis identical to the present one except for slip, enthalpy, and viscosity computations.

## D. General Results

Figures 3 and 4 illustrate typical convergence behavior of the solutions with increasing number of iterations. Figure 3 shows the successive computed nondimensional temperature profiles, including the initial guess, along the stagnation streamline of a sphere with noncatalytic wall. The temperature profiles did not quite converge by the final iteration (2000), so it was necessary to make a rerun using the output of run No. 1 as input data for run No. 2. The figure illustrates that extremely accurate initial guesses are not generally required to achieve convergence; however, the more accurate the initial guess, the faster the solutions will converge. At very large Reynolds numbers, where gradients in flow properties are large, more accurate initial guesses are required to avoid divergence. Figure 4 shows the convergence behavior of wall heat transfer coefficient,  $C_H$ , for the same run. The converged value of  $C_H$ , obtained in run No. 2, is shown at the right margin.

Figure 5a gives the computed velocity components, and the pressure, temperature, and density profiles along the stagnation streamline of a sphere with noncatalytic wall at relatively high freestream Reynolds number (1315). Figure 5b gives the mass fraction profiles for the same run. This run is approaching the maximum Reynolds number for which the program will converge easily without extremely accurate input data or modifications to the program such as spacing the computation increments closer together. The meaning of the quantities in Figure 5a are given in equations (18) through (24); e.g.  $P_1$  is the pressure on the stagnation streamline, and  $P_2$  gives the correction in pressure for small angles away from the stagnation streamline. A thick "shock wave" is indicated by the steep gradients in the region  $1.06 < r < 1.12$ .

The same quantities presented in Figure 5 are plotted in Figure 6 for a smaller Reynolds number (185). Figure 6a reveals that all evidence of a shock wave has disappeared at this low Reynolds number. Comparison of Figures 5b and 6b shows that the degree of dissociation and ionization decreases drastically in lower density flow.

Figure 7 gives the flow profiles along the stagnation streamline of a sphere with fully catalytic wall at a medium Reynolds number (458). The Sutherland viscosity formula was used because this run was made before the

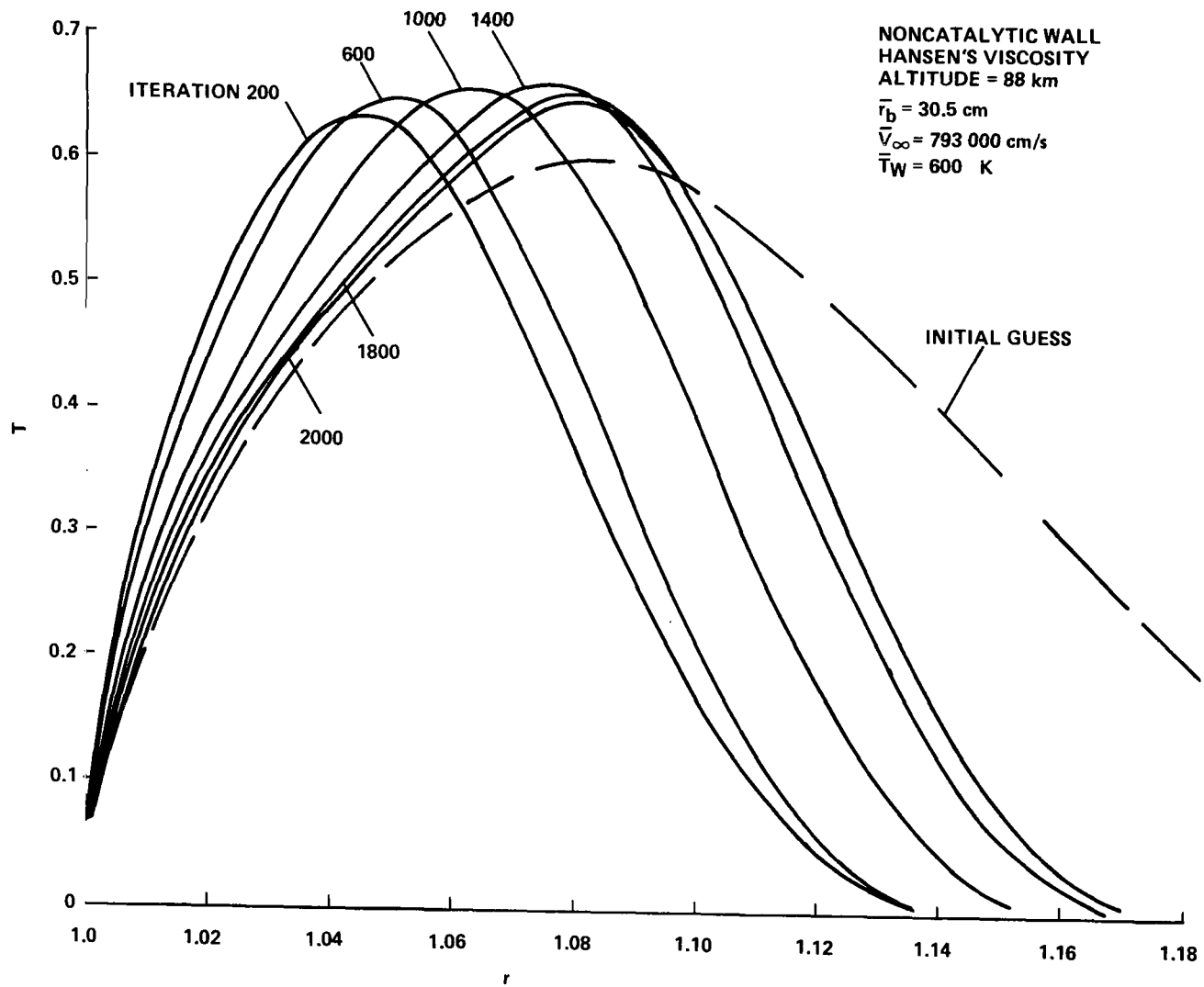


Figure 3. Convergence of temperature profiles for sphere with increasing number of iterations.

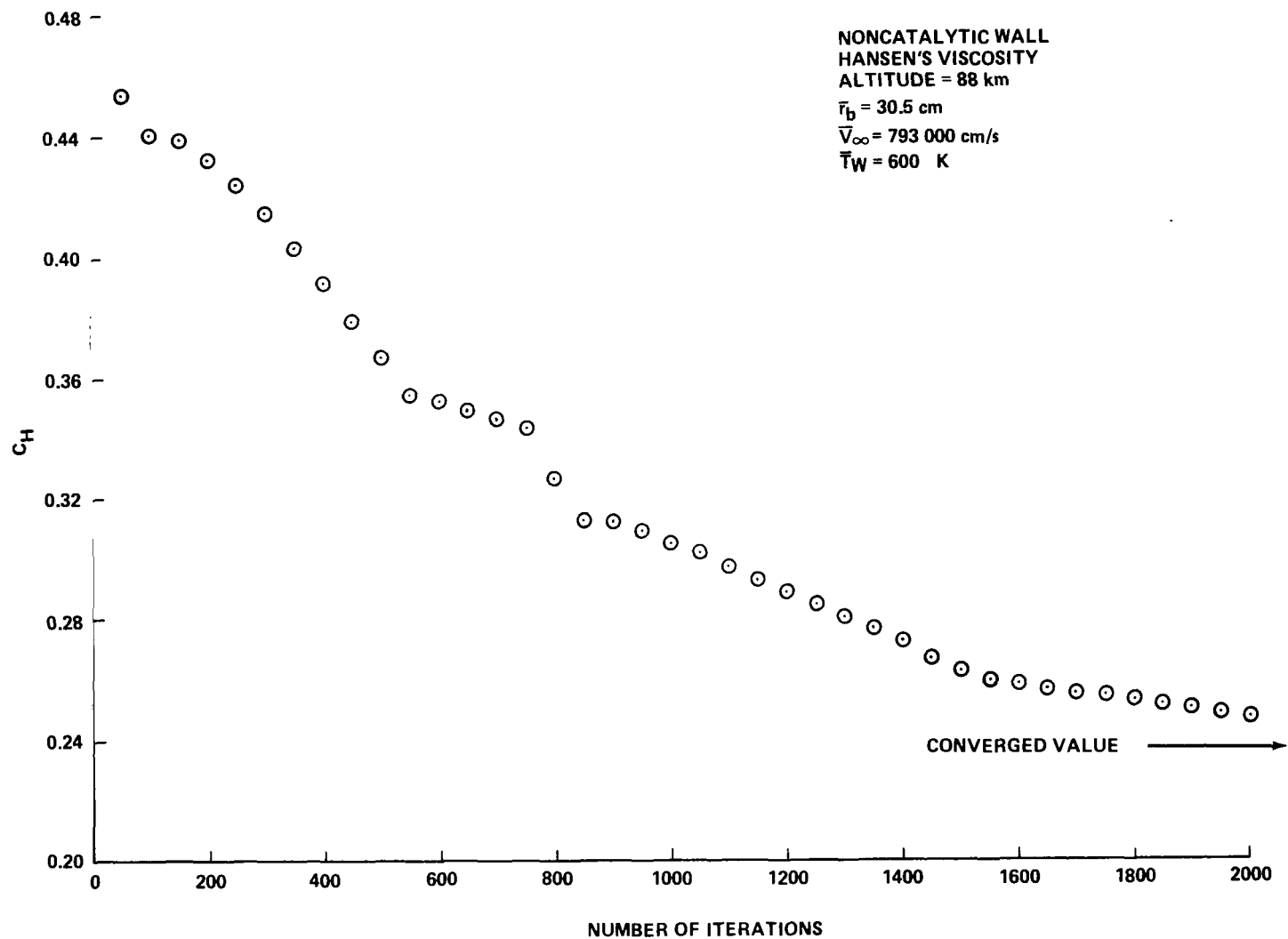
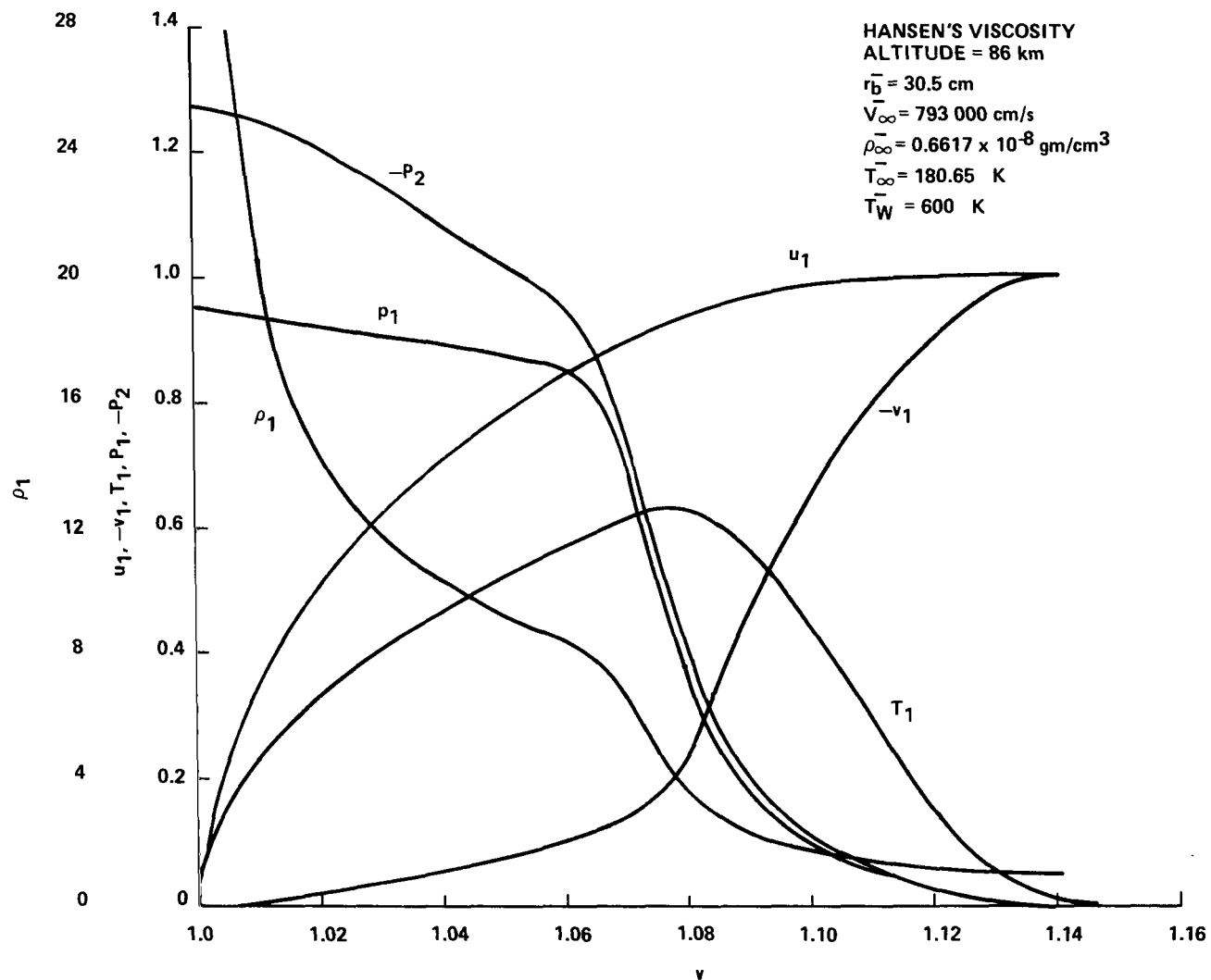
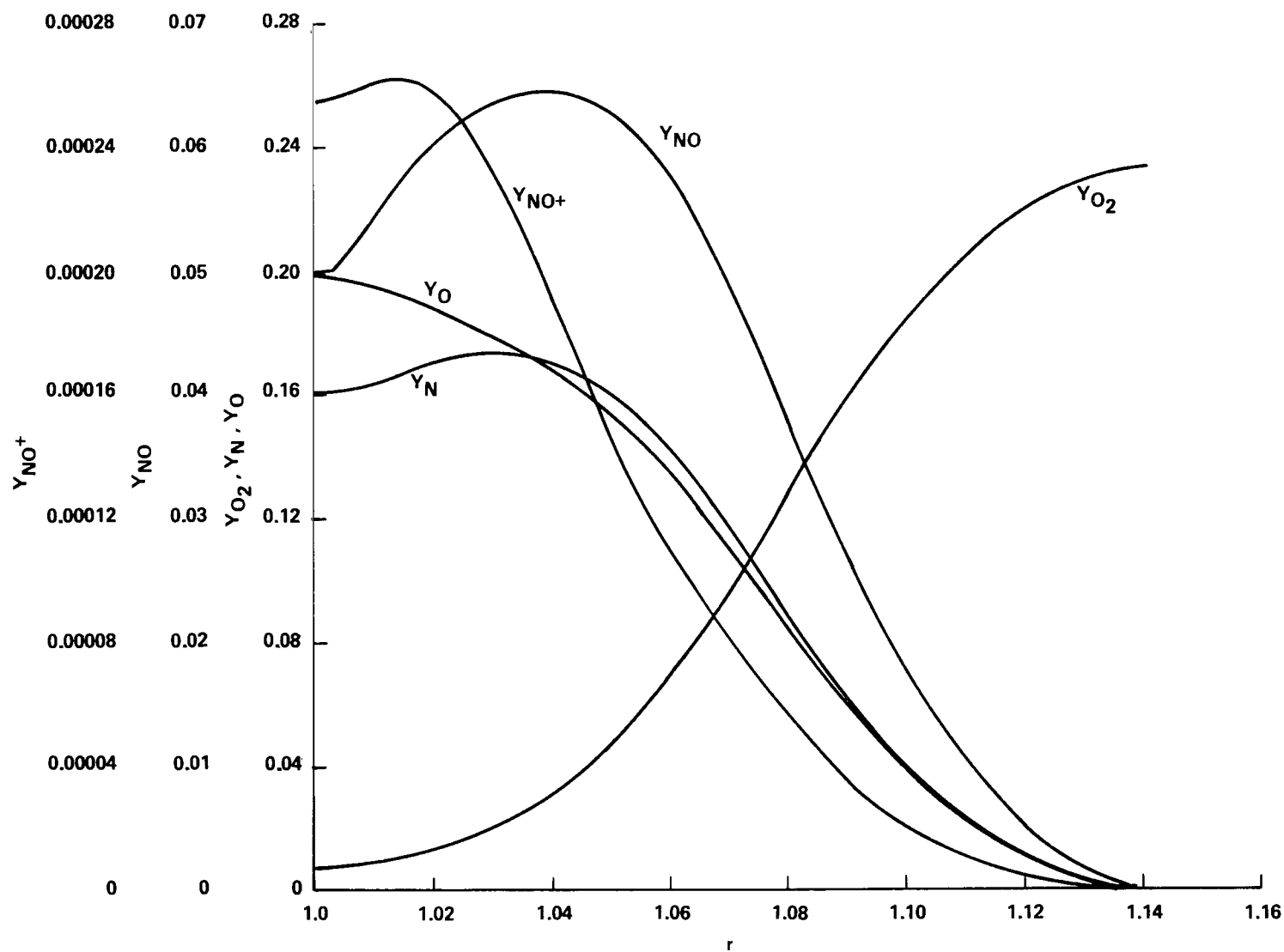


Figure 4. Typical convergence of stagnation point heat transfer coefficient for sphere with increasing number of iterations.



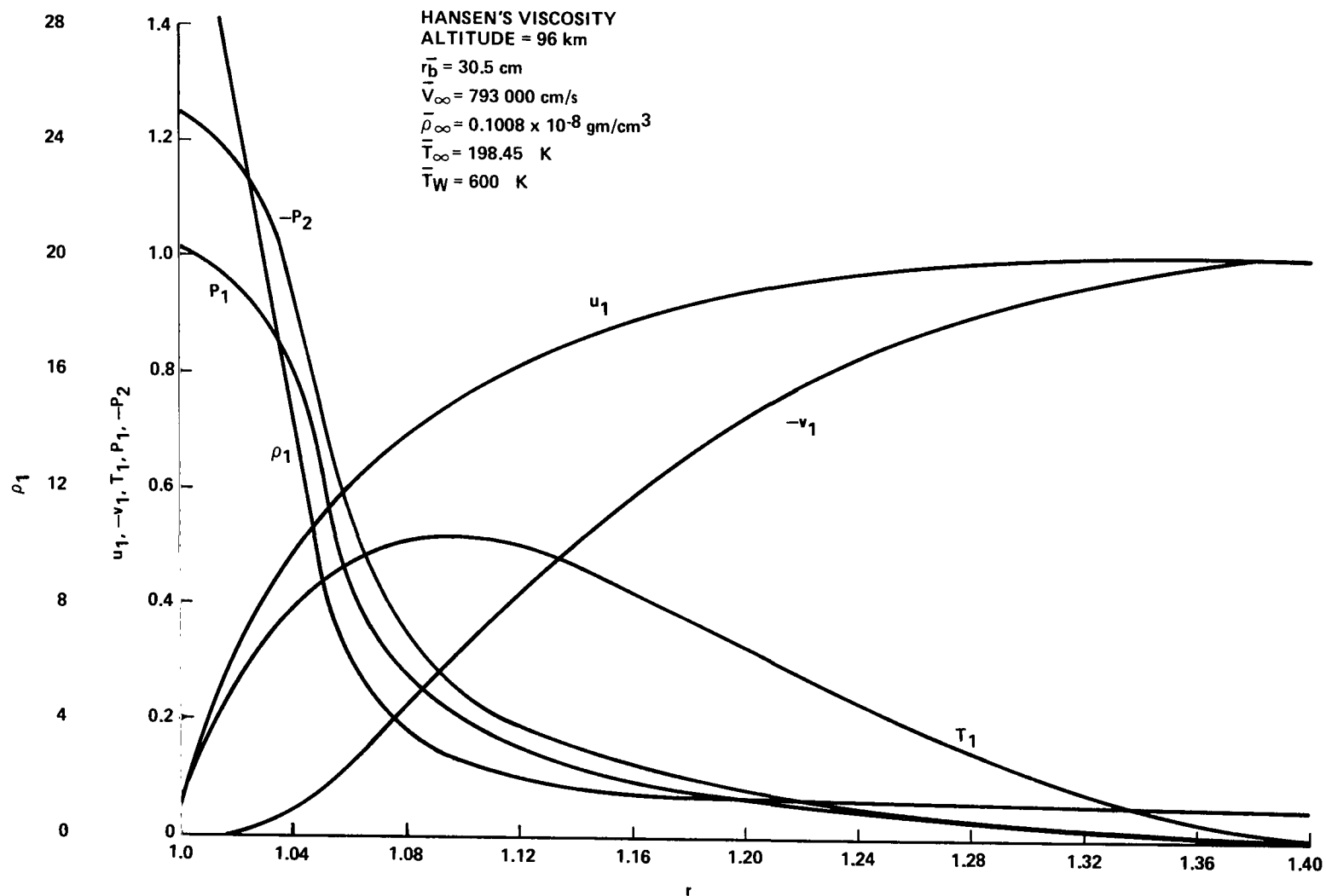
a. Velocity and thermodynamic property profiles.

Figure 5. Flow profiles for sphere with noncatalytic wall at large Reynolds number ( $Re_\infty = 1315$ ).



b. Mass fraction profiles.

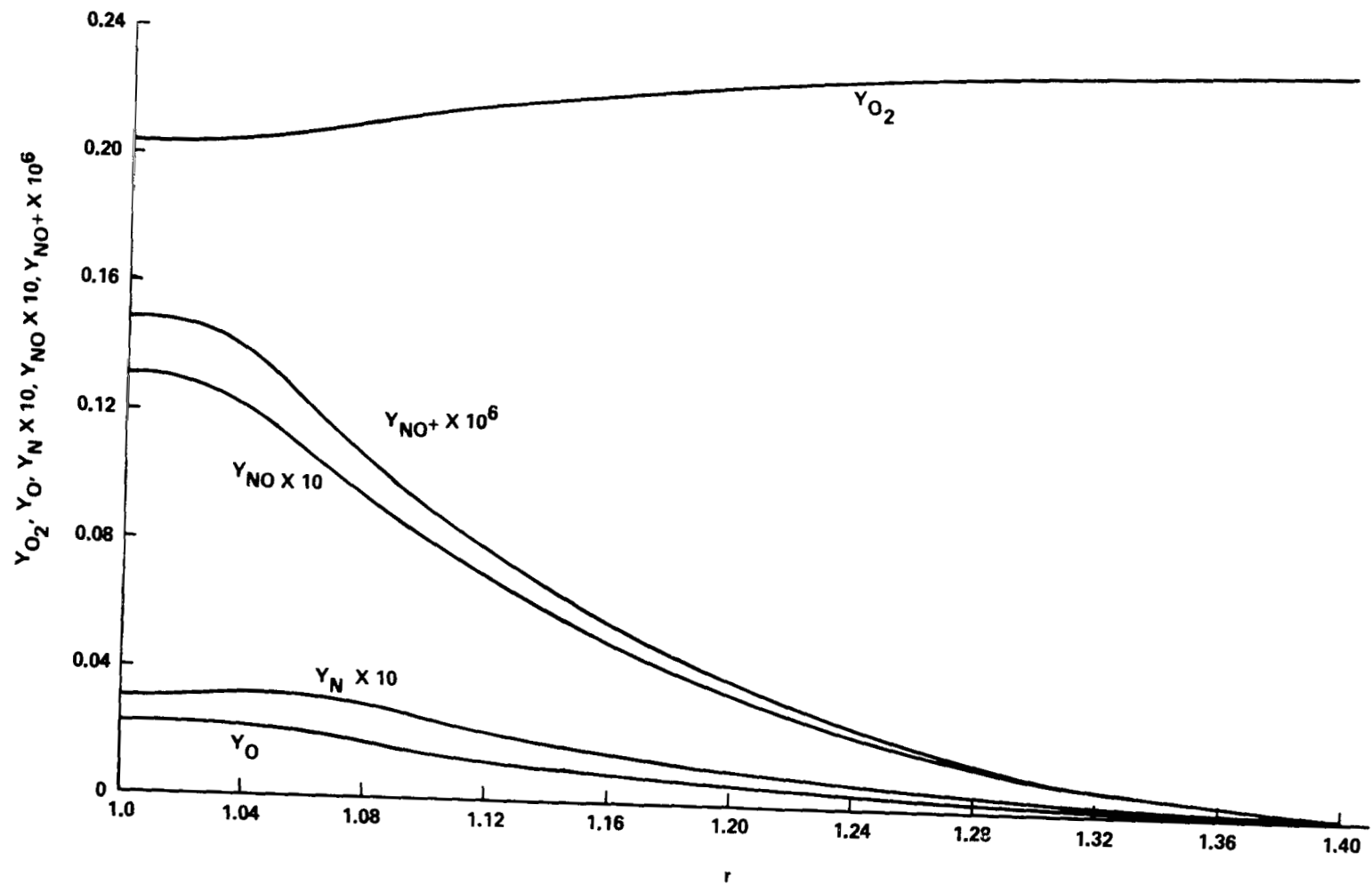
Figure 5. (Concluded)



a. Velocity and thermodynamic property profiles.

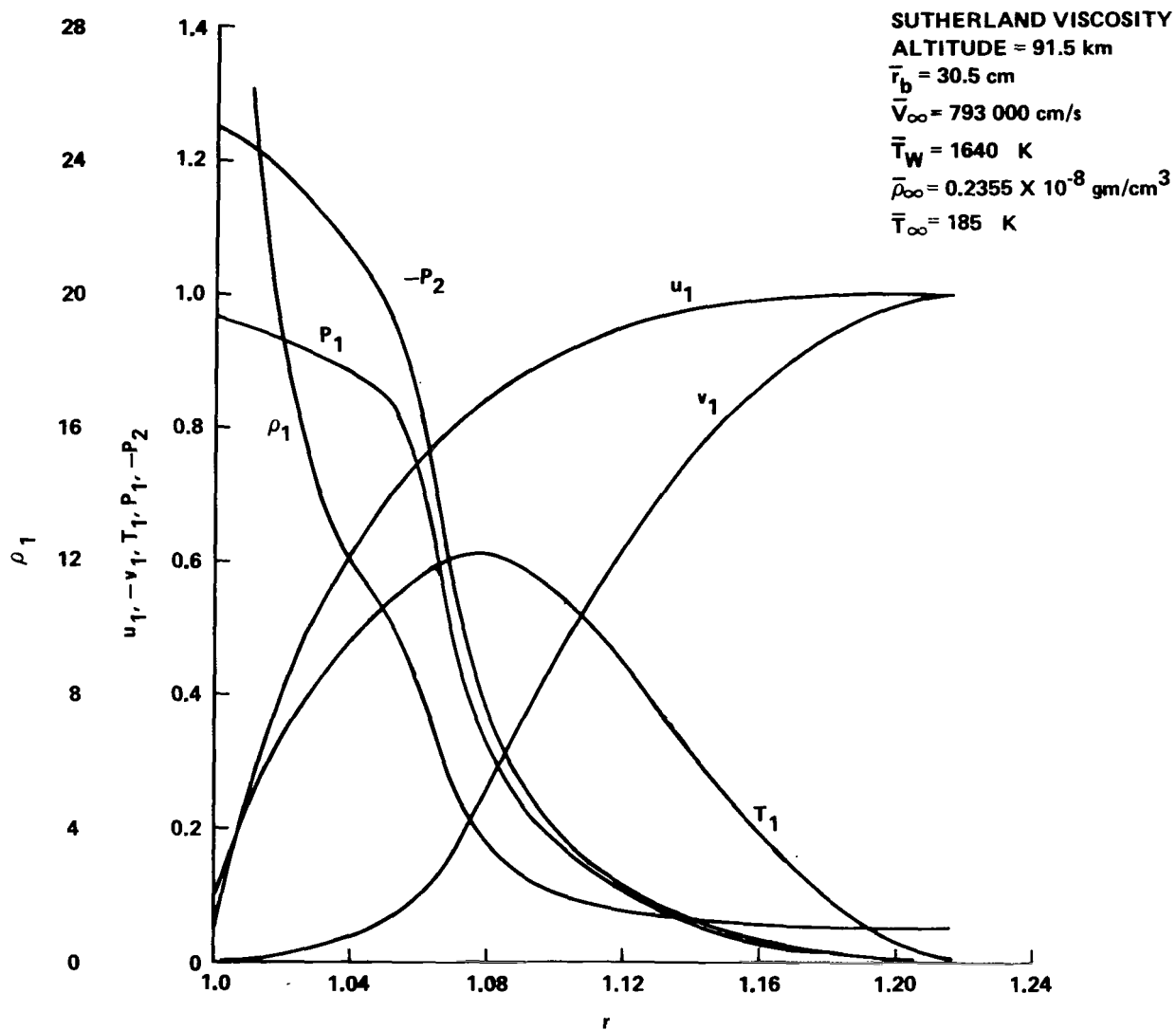
Figure 6. Flow profiles for sphere with noncatalytic wall at small Reynolds number ( $Re_\infty = 185$ ).





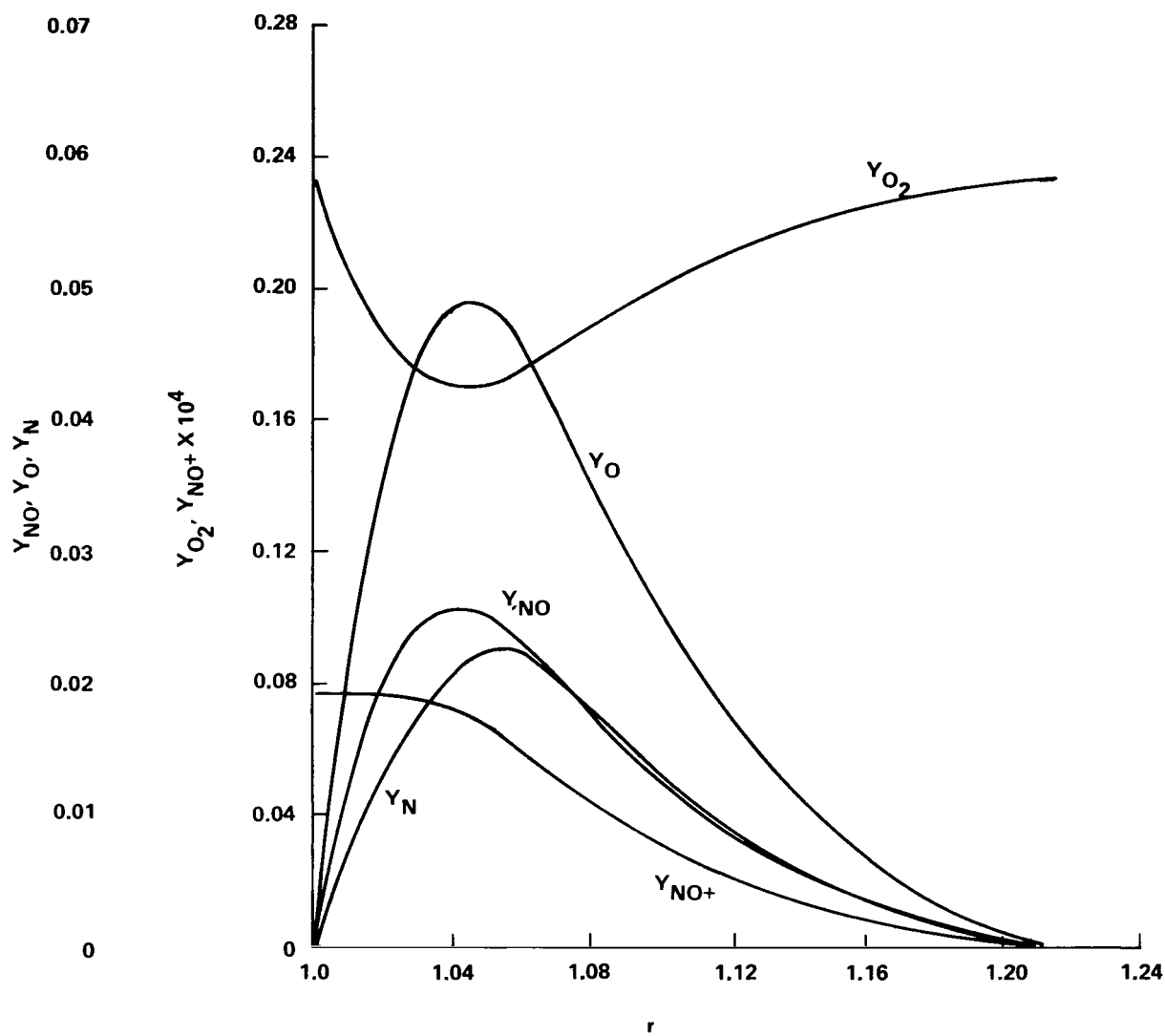
b. Mass fraction profiles.

Figure 6. (Concluded)



a. Velocity and thermodynamic property profiles.

Figure 7. Flow profiles for sphere with fully catalytic wall ( $Re_\infty = 458$ ).



b. Mass fraction profiles.

Figure 7. (Concluded)

viscosity modification to the program. The density profile gives a hint of a shock wave similar to that shown in Figure 5a. Comparison of Figures 5b, 6b, and 7b illustrate the difference in species mass fraction profiles between a noncatalytic wall and a fully catalytic wall. The species mass fractions approach their freestream values at the catalytic wall except for the ions  $\text{NO}^+$  and  $\text{e}^-$ . The wall is assumed noncatalytic with respect to ion recombination in this program.

Figures 5 through 7 provide useful starting data for running the program at conditions near the ones in these figures.

Figure 8 compares temperature, density, and velocity profiles for a sphere and cylinder at the same flow conditions as in Figure 5. The profiles are remarkably similar. Therefore, the profiles for a sphere, Figures 5, 6, and 7, are adequate to use as starting data for the cylindrical option of the program.

The change in the stagnation line temperature profile with freestream density, or  $\text{Re}_\infty$ , is illustrated in Figure 9. The peak value in nondimensional temperature decreases and the shock layer thickness increases with increasing altitude or decreasing density.

The freestream density and speed greatly affect the degree of dissociation of the air molecules and, hence, the viscosity of the gas mixture. Figure 10 gives the peak values from the viscosity and mass fraction profiles for N, O, and NO as a function of altitude for a fixed large value of freestream speed. As before, the freestream thermodynamic properties associated with altitude are obtained from Reference 14. Although the stagnation enthalpy is large, the figure shows that dissociation becomes negligible and viscosity approaches the Sutherland value at altitudes greater than approximately 100 km. This is due to the decreased reaction rates at the lower temperatures which occur in the shock layer at higher altitudes (Fig. 9). Figure 11 illustrates that dissociation and the viscosity ratio increase with increasing freestream speed at the fixed altitude of 96 km.

A parametric study was made to determine the effect of altitude (or density), freestream speed, and wall temperature on the stagnation point heat transfer coefficient,  $C_H$ . Figure 12 gives  $C_H$  for a sphere and cylinder as a function of altitude with fixed freestream speed and wall temperature. Hansen's viscosity, equation (48), was used for the sphere, but Sutherland's viscosity was used for the cylinder because those runs were made before the program was modified to compute Hansen's viscosity. The figure shows a steady increase

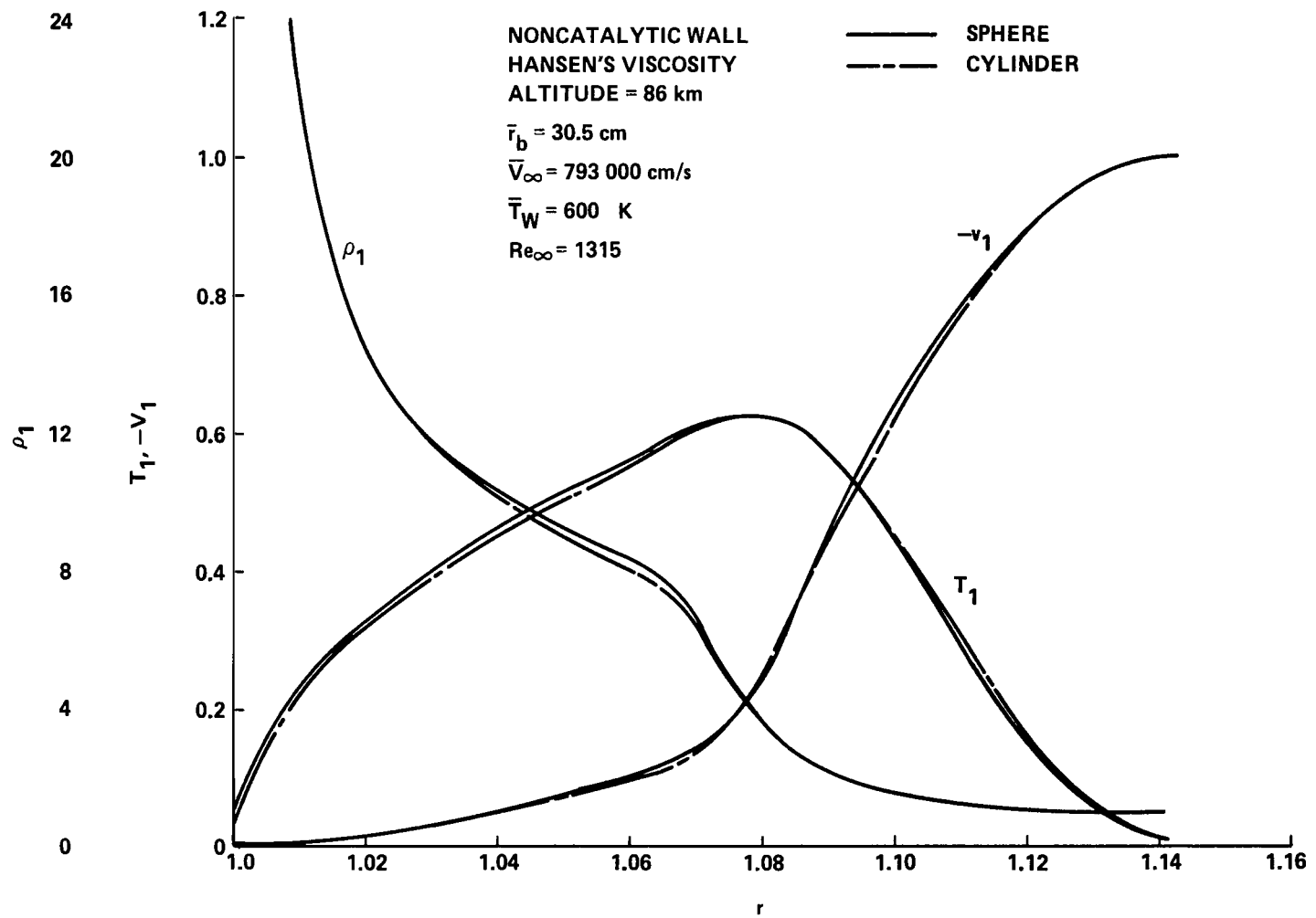


Figure 8. Comparison of flow profiles for sphere and cylinder.

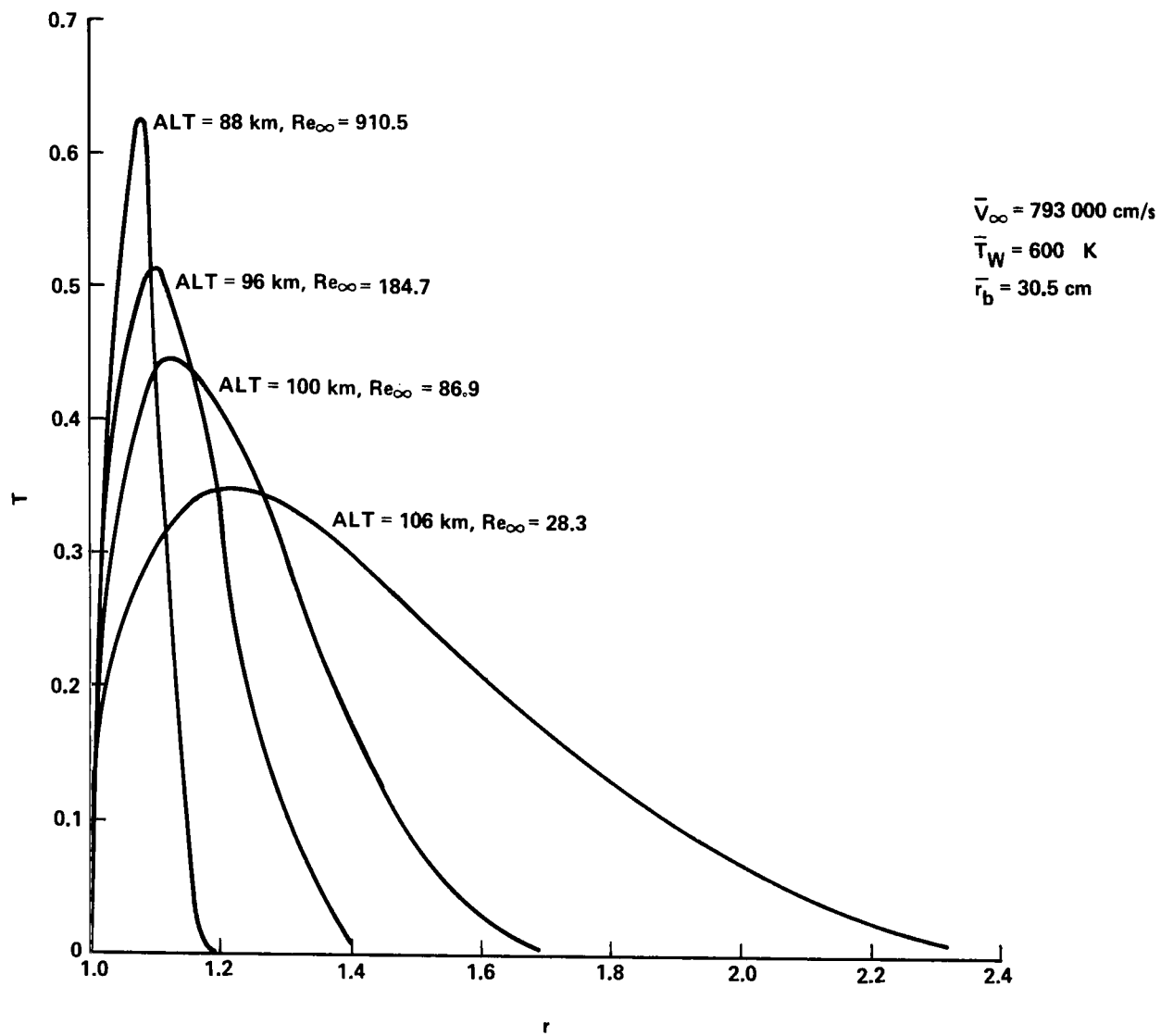


Figure 9. Temperature profiles at various Reynolds numbers for sphere.

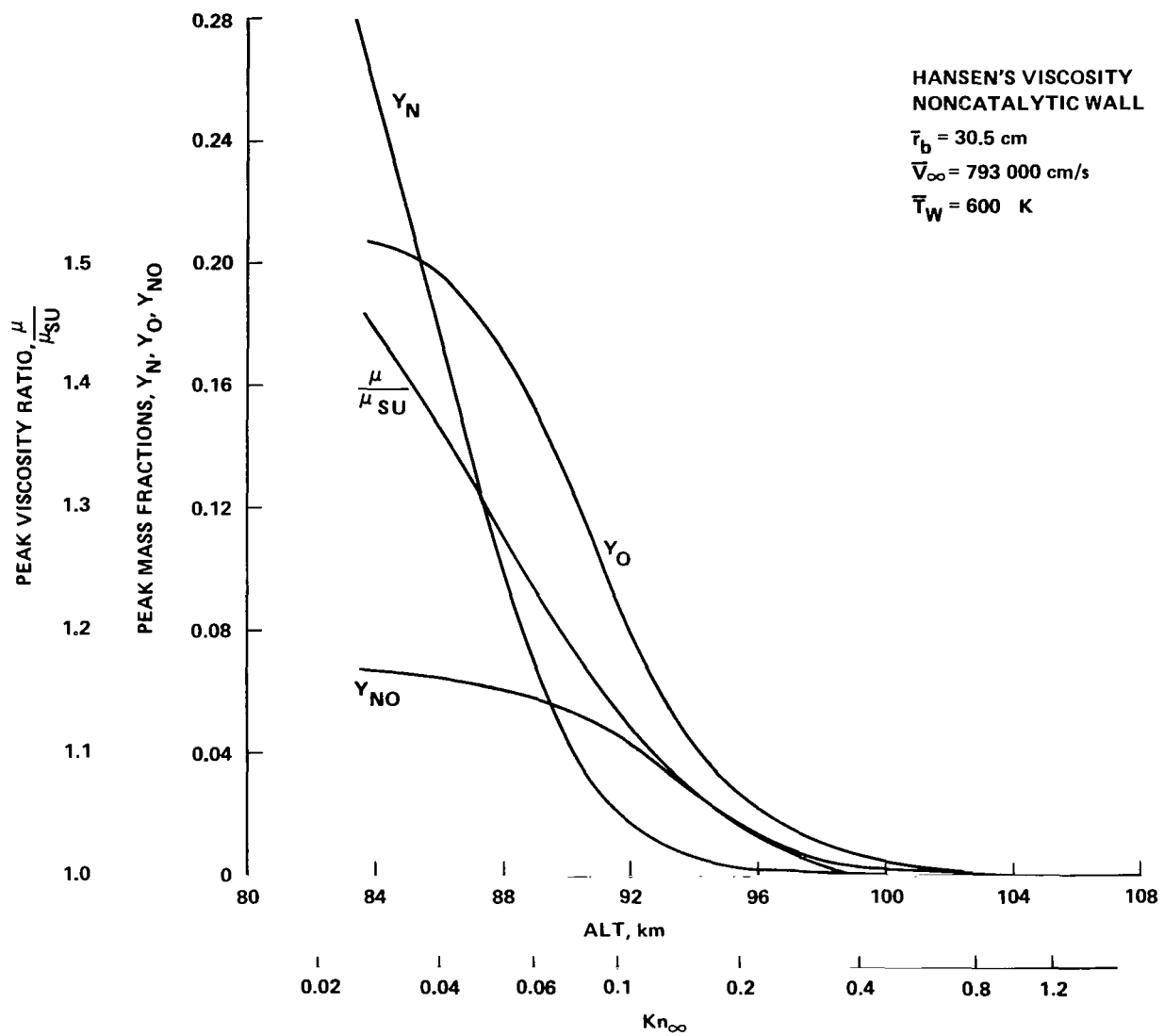


Figure 10. Effect of variation in altitude on dissociation and viscosity in shock layer of sphere.

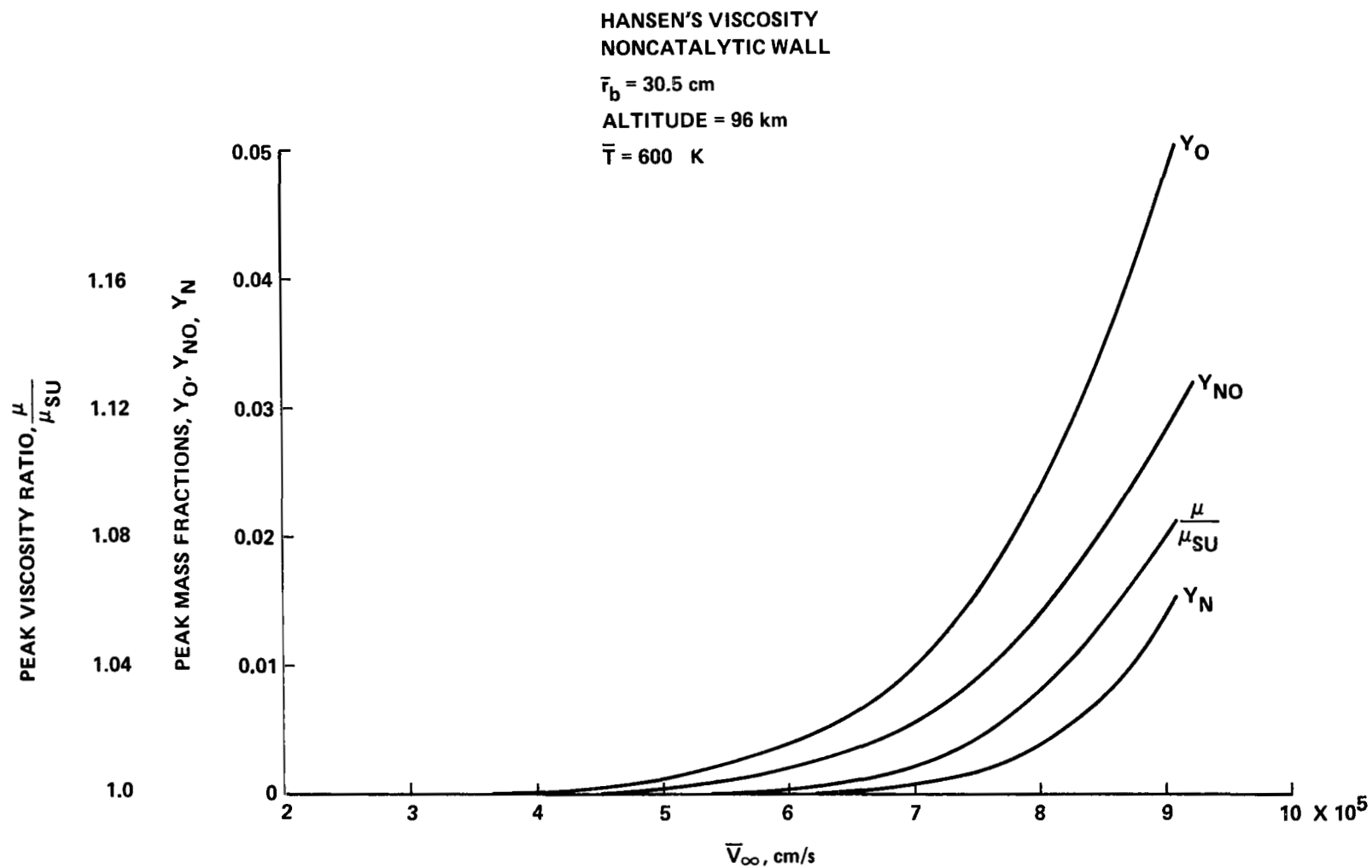


Figure 11. Effect of freestream speed on dissociation and viscosity in shock layer of sphere.



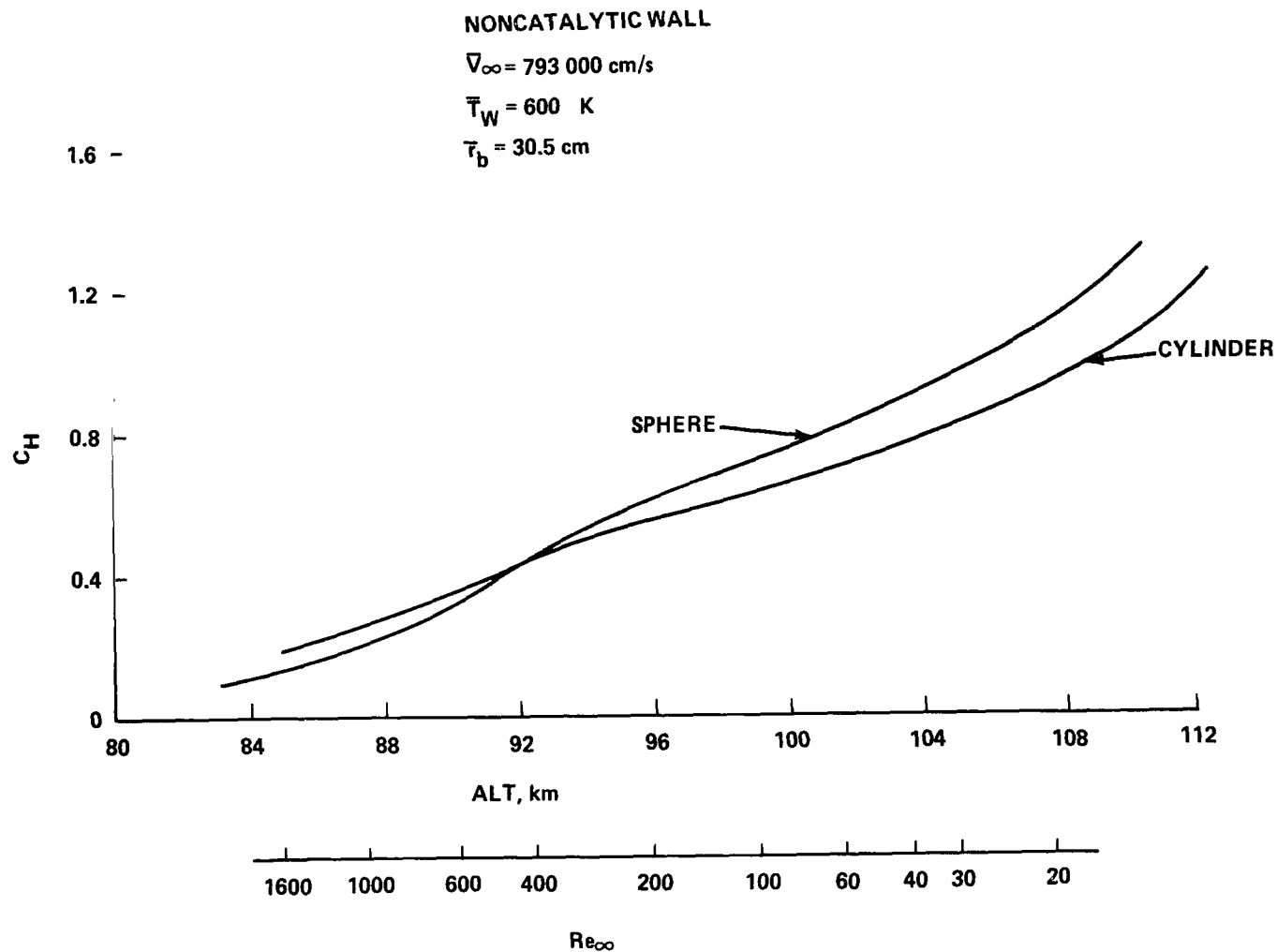


Figure 12. Stagnation point heat transfer coefficients for spheres and cylinders as a function of altitude.

in  $C_H$  with increasing altitude. The theoretical free molecular value ( $C_H = 1.0$ ) is exceeded for the sphere at approximately 106 km and for the cylinder at approximately 109 km. The curves continue to climb with increasing slope at higher altitudes. This unrealistic behavior for a fixed wall temperature might be attributed to a breakdown in the continuum approach (Navier-Stokes equations with slip boundary conditions) at extremely high altitudes and low densities. In particular, the assumption of a thin Knudsen layer, which is implicit in the present analysis, becomes invalid at very low densities. One must also consider the practicality of a specified  $T_w$  in rarefied flow. This breakdown is gradual, and one cannot pinpoint a sharp boundary beyond which the program gives invalid results. However, the results should be treated with increasing skepticism, for this body size and flow conditions, at altitudes greater than approximately 104 km, or  $Re_\infty < 40$ .

An interesting feature of Figure 12 is the crossover of the curves for the sphere and cylinder. This crossover should not be attributed to the differences in viscosity computation because the effects of Hansen's viscosity (used for the sphere but not for the cylinder) is to increase  $C_H$  at low altitudes but not to affect  $C_H$  at high altitudes where dissociation is negligible. Therefore, if Hansen's viscosity were also used for the cylinder, the crossover of the two curves should be expected at a higher altitude. The explanation for this crossover is a result of the stronger merged shock layer effect on the sphere.

The variation of  $C_H$  with freestream speed for a sphere at fixed altitude (96 km) and wall temperature (600 K) is shown in Figure 13. The effect of increasing freestream speed, or stagnation enthalpy, is to decrease  $C_H$ .

Figure 14 gives the variation in  $C_H$  with wall temperature for a sphere at fixed altitude (96 km) and speed (793 000 cm/sec). An increase in wall temperature produces an increase in  $C_H$ .

The data in Figures 12 through 14 are replotted in Figure 15 as a function of  $K^2$ ; the similarity parameter is given by equation (62). This parameter includes all the independent variables in Figures 12, 13, and 14, but the effect of freestream density is obviously dominant in determining the value of  $K^2$  and  $C_H$ . The data were also plotted as a function of other similarity parameters (not shown), but  $K^2$  correlated the data as well as, or better than, any of the other parameters. This is not the case if wall catalysis is varied. Therefore,  $K^2$  was selected as the similarity parameter to use in presenting the remainder of the data.

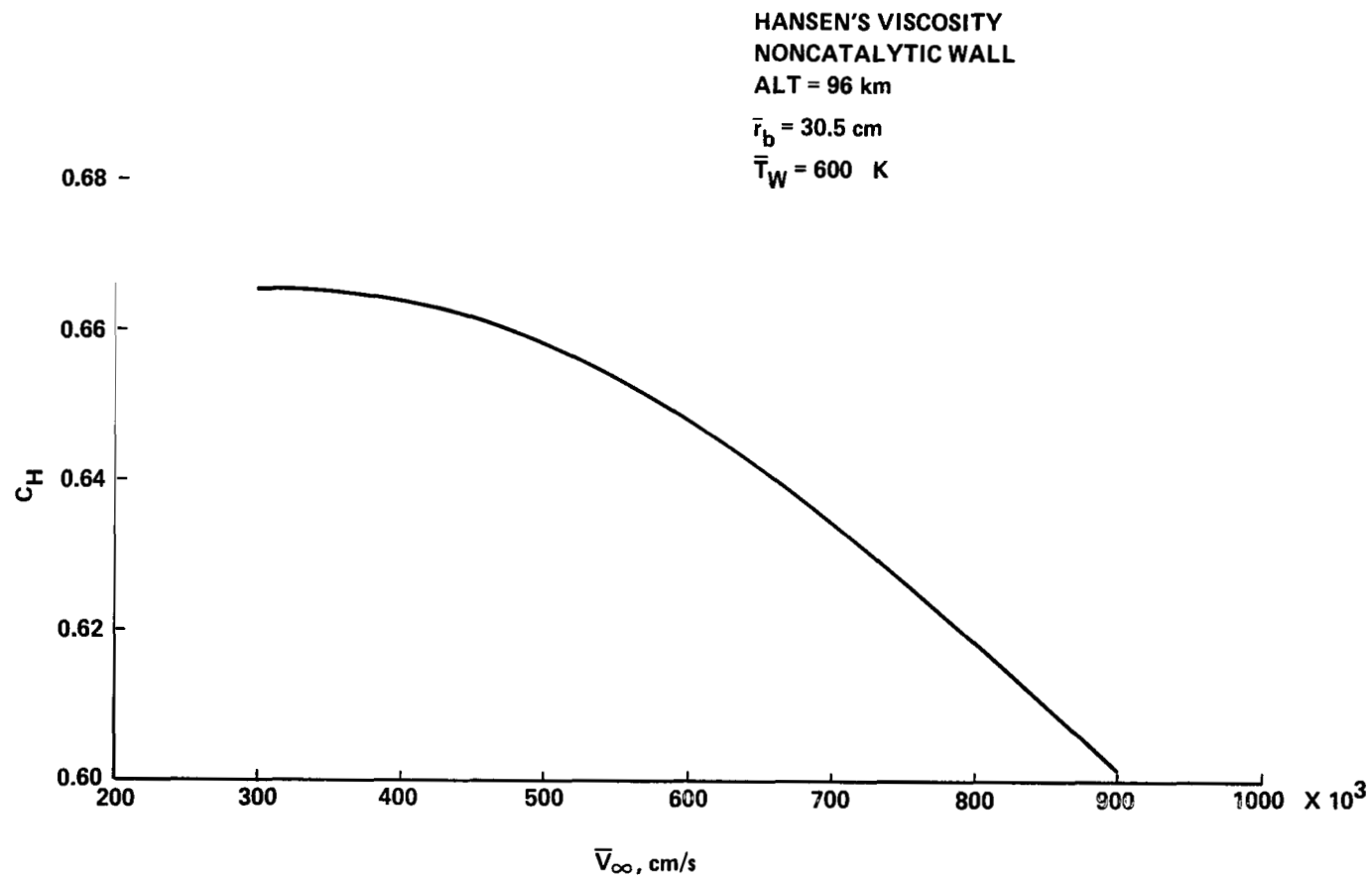


Figure 13. Stagnation point heat transfer coefficient for sphere as function of freestream speed.

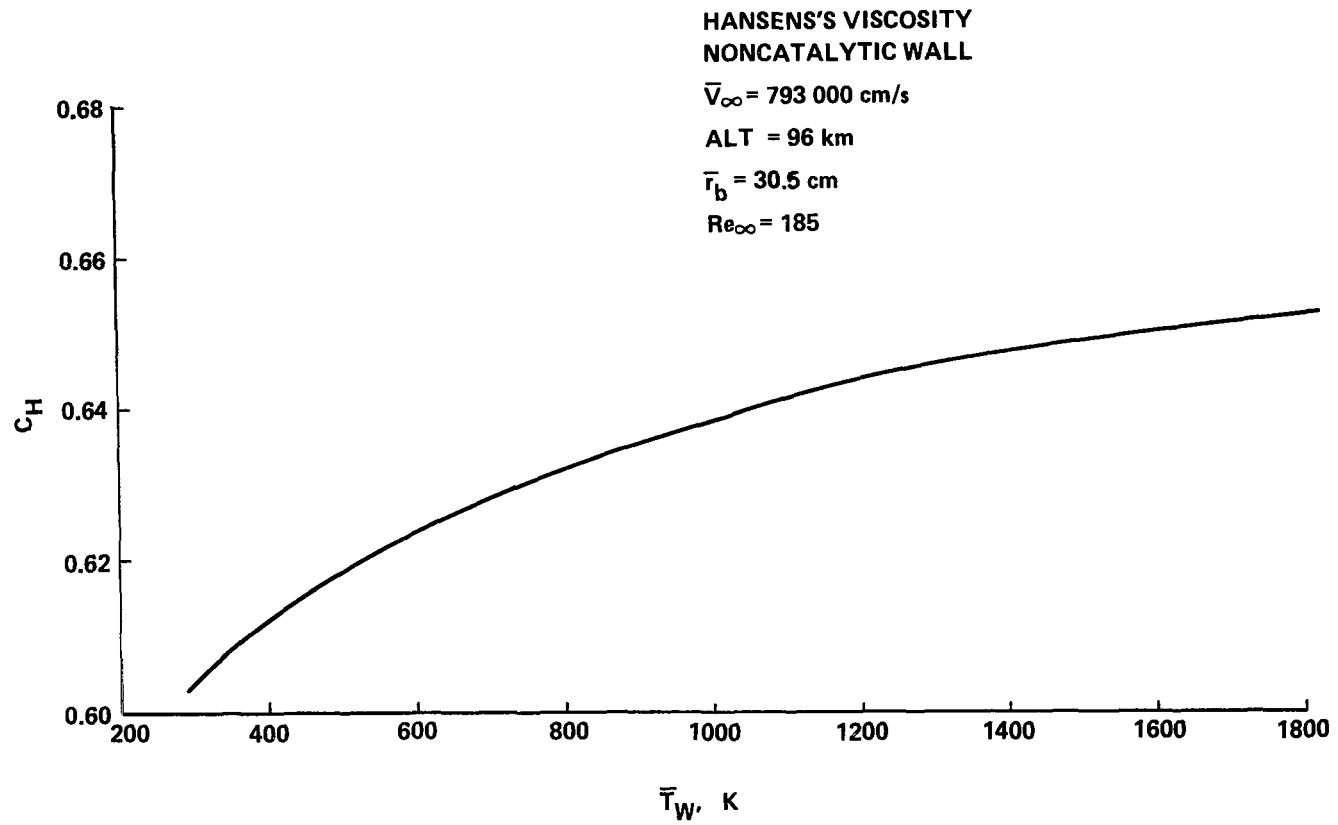


Figure 14. Stagnation point heat transfer coefficient for sphere as function of wall temperature.

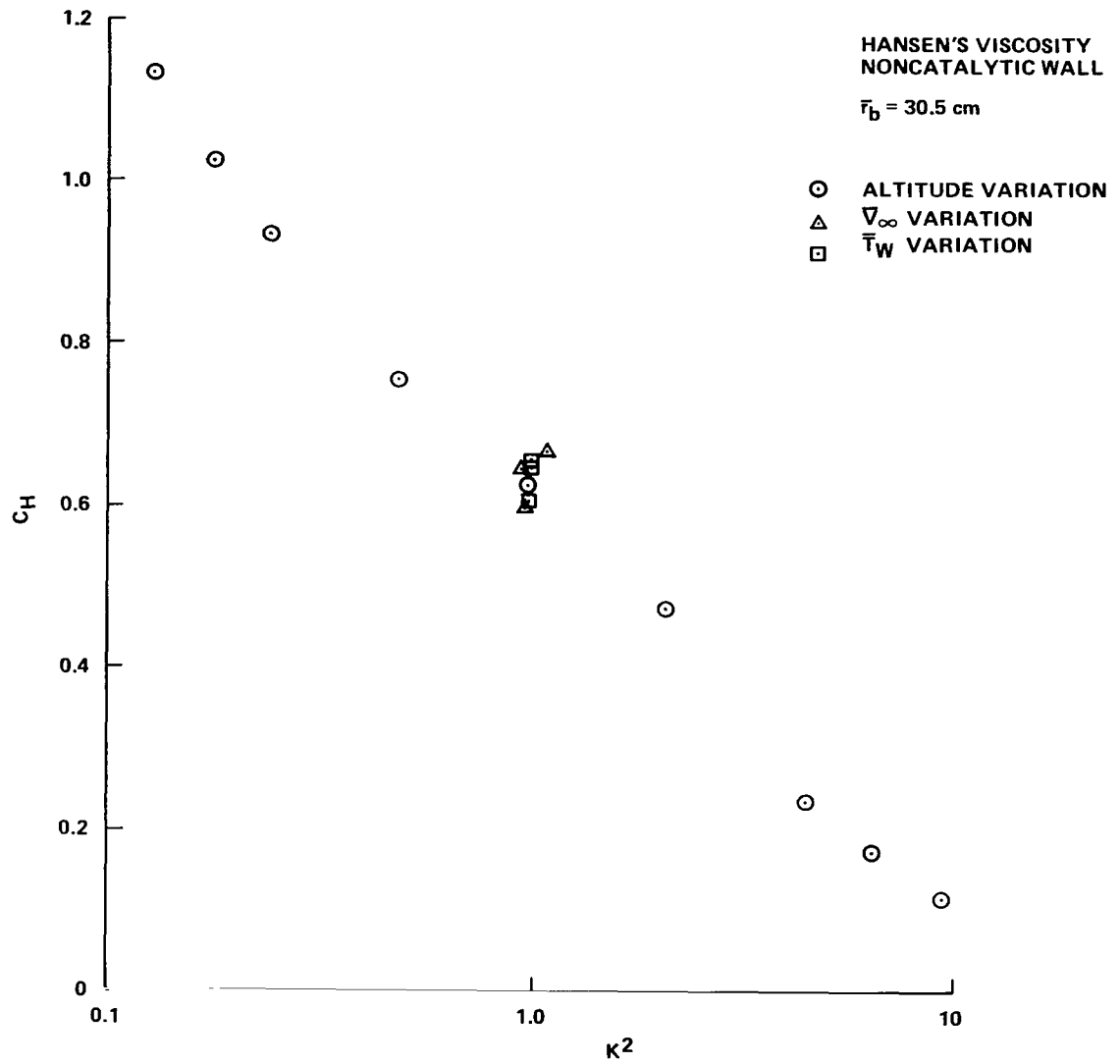


Figure 15. Stagnation point heat transfer coefficient for sphere as function of  $K^2$ .

Figure 16 gives the nondimensional shock layer thickness,  $n_\infty$ , for spheres and cylinders as a function of  $K^2$ . The shock layer is thin at the continuum end (large  $K^2$ ) of the flow regime and becomes very thick at highly rarefied conditions (small  $K^2$ ).

Figure 17 presents the computed slip speed for a sphere and cylinder as a function of  $K^2$ . The slip speed is computed at the outer edge of the Knudsen layer, but it is applied at the wall in the outer flow solution, i.e., a thin Knudsen layer is assumed. This is a source of error at high altitudes where the Knudsen layer thickness becomes significant. The slip speed is greatly affected by variation in freestream speed and wall temperature. However, these effects are not correlated well by the parameter  $K^2$  as shown for the sphere by the partially filled symbols. To estimate the slip speed for given conditions, one can extrapolate from the solid curves using the partially filled symbols as a guide in correcting for freestream speed and wall temperature. Once again it appears that the proper  $T_W$  must be used for rarefied flow calculations. The decrease in slip speed with increasing altitude (decreasing  $K^2$ ) at high altitude (small  $K^2$ ) seems unrealistic and has been criticized. However, decreasing slip speed decreases the  $C_H$  computed by this program. Since the computed  $C_H$  is too large at high altitudes for a fixed  $T_W$  (Figs. 12 and 15), the decrease in slip speed at high altitudes tends to compensate for the overprediction of  $C_H$  and is, from a practical standpoint, a favorable phenomenon.

The temperature at the edge of the Knudsen layer is given as a function of  $K^2$  in Figure 18. This temperature also depends strongly on freestream speed and wall temperature.

The information in Figures 15 through 18 is useful as input data in running the computer program (Appendix A).

An example of the practical use of this computer program is given in Figure 19. Stagnation point heating rates for the spherical nose of the Space Shuttle External Tank are given as a function of trajectory time. The solid curve is the heating rate predicted by the MSFC Thermal Environment Branch (ED33) and is based on theoretical continuum methods at low altitudes and experimental data at high altitudes. The filled diamond symbols were computed using the present program. The trajectory time interval from approximately 275 to 475 sec corresponds to an altitude interval which is too high for this program, i.e., the predicted values of  $C_H$  were unrealistically large. For a

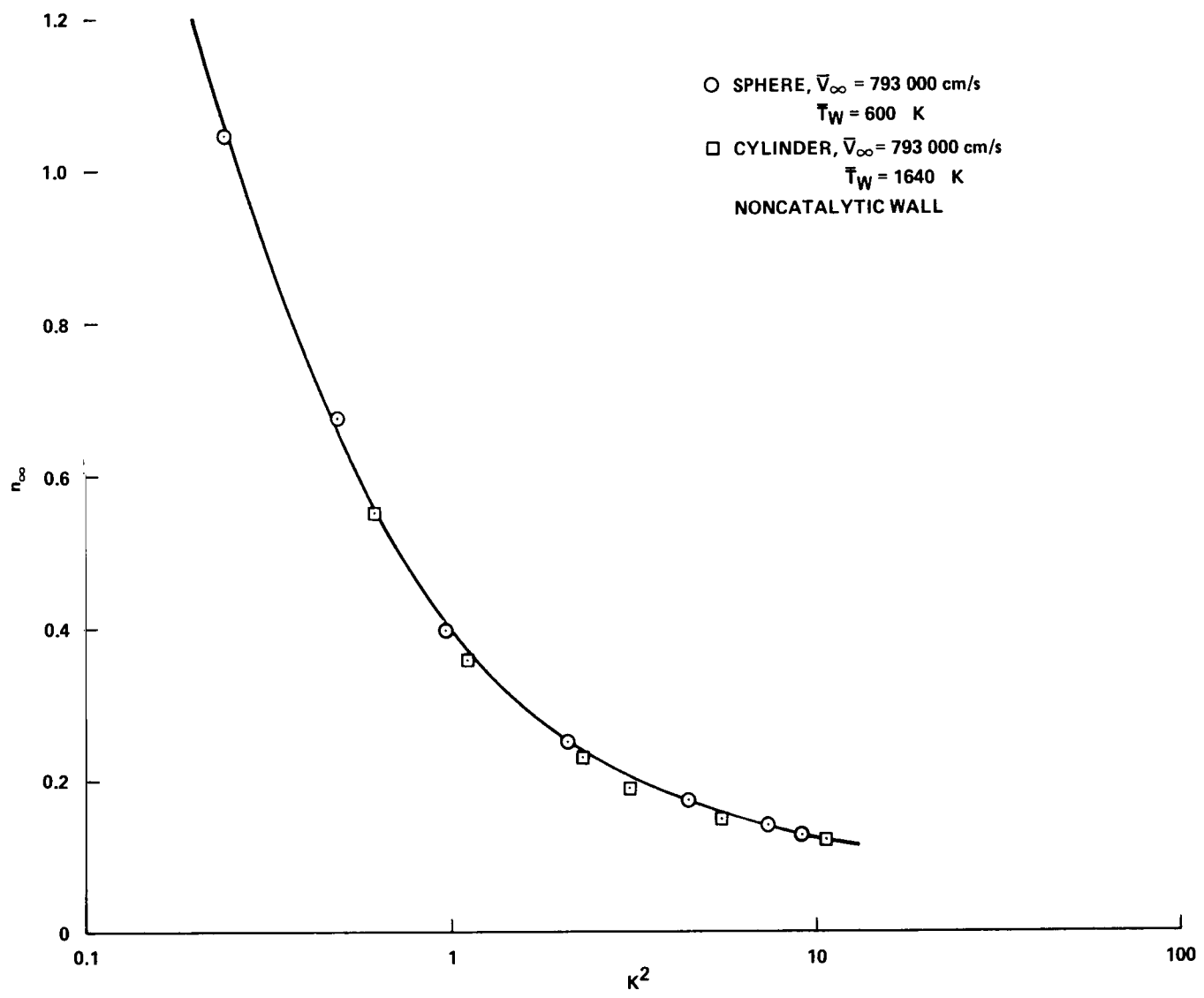


Figure 16. Shock layer thickness for sphere and cylinder.

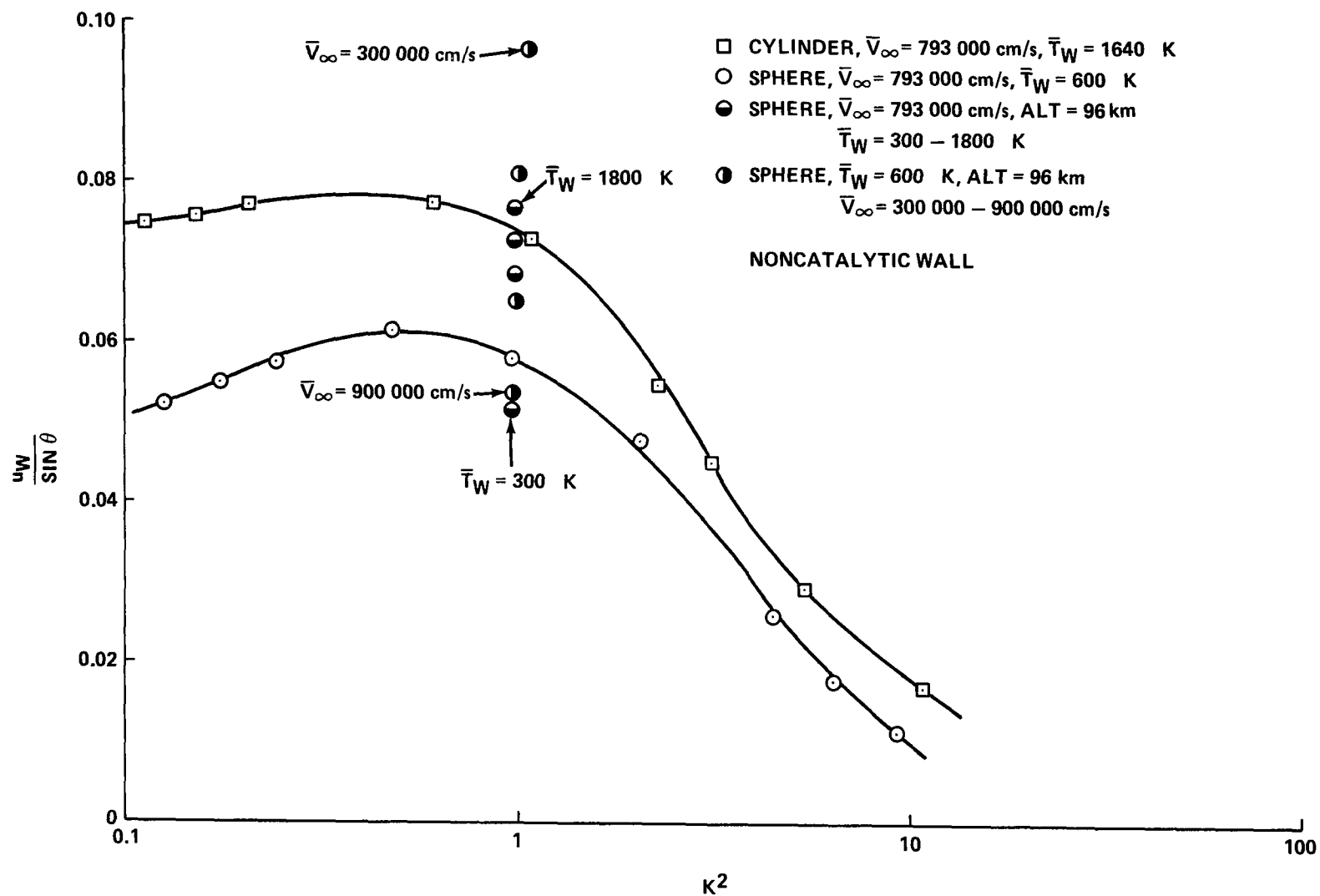


Figure 17. Slip speed at body surface for sphere and cylinder.



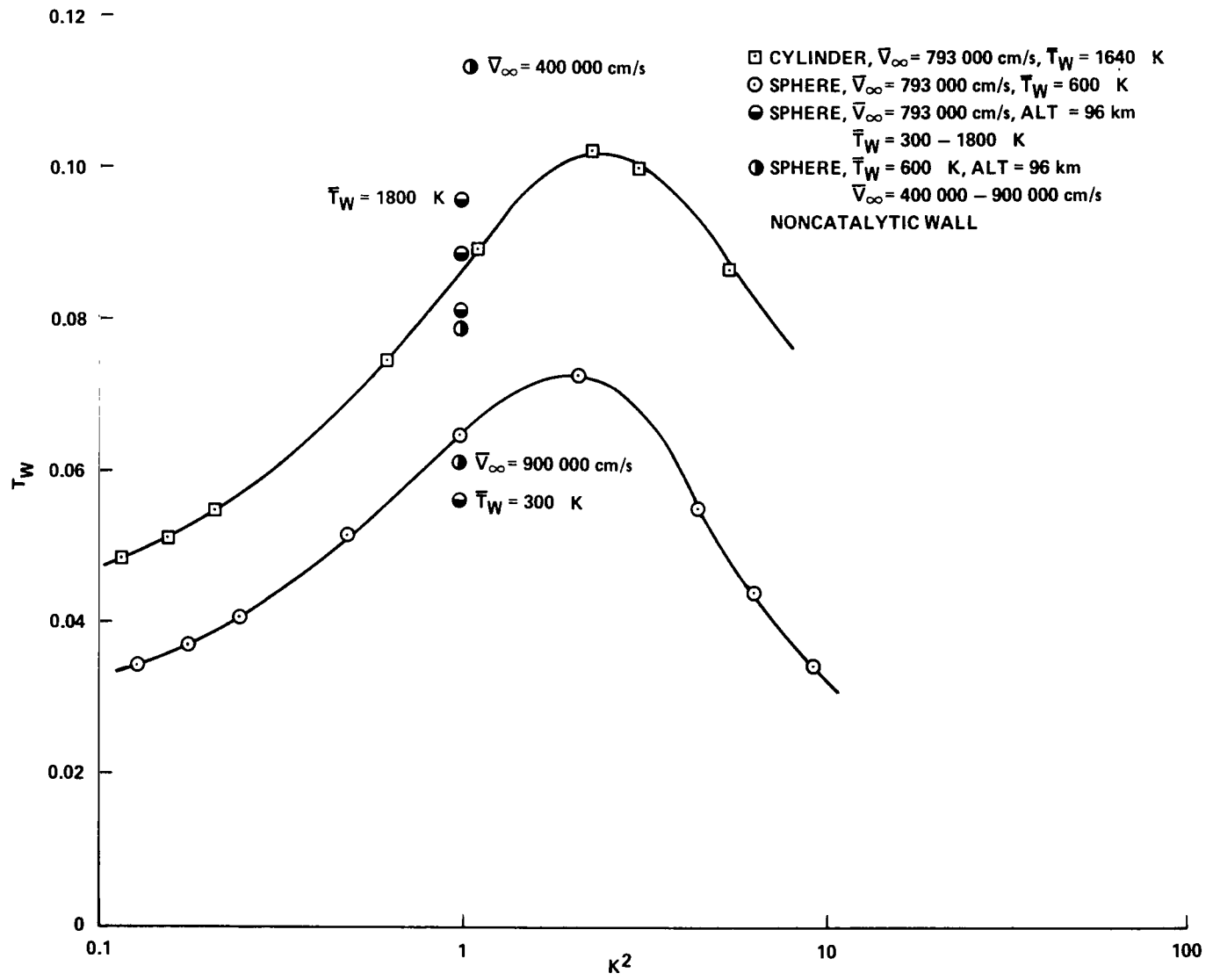


Figure 18. Temperature of air at wall for sphere and cylinder.

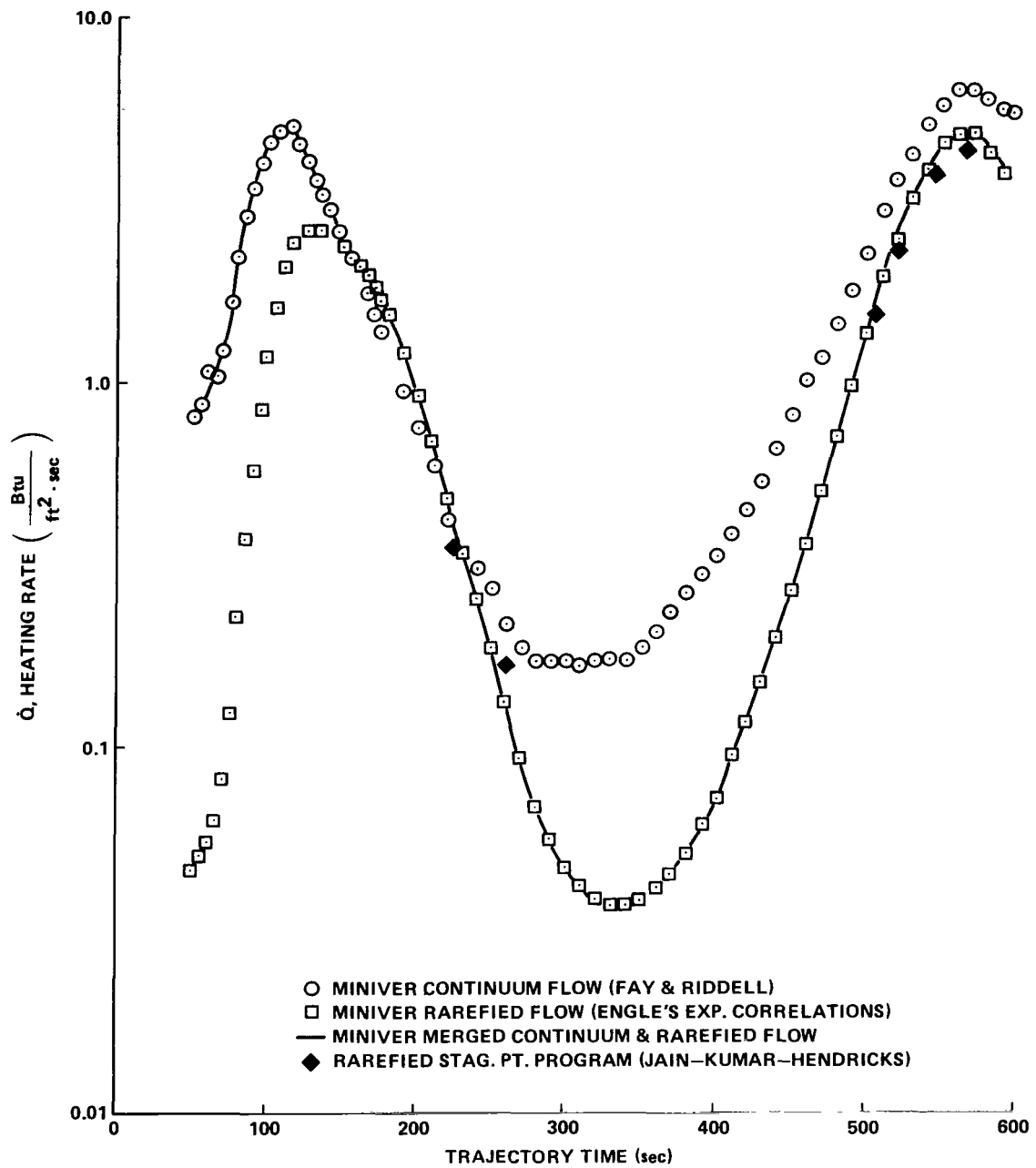


Figure 19. Space Shuttle external tank stagnation point heating rate.

body of this size ( $\bar{r}_b = 30.5$  cm), the altitude interval for which this program is applicable is approximately 86 to 105 km. For this altitude interval, the predicted values from this program agree quite well with the predictions from experimental data for a known  $T_W$ .



## APPENDIX A. COMPUTER PROGRAM

## APPENDIX A. COMPUTER PROGRAM

### General Information

The computer program is written in Fortran IV language, and it is run on the Univac 1108 computer at MSFC. As currently written, the program is limited to a maximum of 2000 iterations. If convergence is not achieved in 2000 iterations, the program is rerun using the output from the first run as starting data for the second run.

The program has three options:

- (1) Body: sphere or two-dimensional circular cylinder
- (2) Wall catalyticity: noncatalytic wall or fully catalytic wall
- (3) Viscosity: Sutherland or Hansen's high temperature model.

The choices made in options (1) and (2) make very little difference in the computer time used. However, the Hansen viscosity option takes significantly more computer time than the Sutherland option; e.g., if the full 2000 iterations are used, the Sutherland option takes approximately 8 min and the Hansen option takes about 10 min.

### Program Input

The quantities needed for input to the program are defined and their functions described as follows:

Computer Program Variable	Symbol Used in Report	Dimensions	Description	Suggested Source or Range of Values
EFFR1	$n_{\infty}$	Dimensionless	Initial guess for nondimensional shock layer thickness	Figure 16
TOL		Dimensionless	Computation is stopped when $C_H$ converges within a prescribed tolerance. If $ C_{H_N} - C_{H_{N-1}}  < \text{TOL}$ , stop program and print results	0.00001 - 0.0001
EPSI		Dimensionless	Maximum change allowed in all computed dimensional quantities from one iteration to the next	0.001 - 0.005

Computer Program Variable	Symbol Used in Report	Dimensions	Description	Suggested Source or Range of Values
ALT		km	Altitude corresponding to the freestream thermodynamic properties (used for identification only)	
RADB	$\bar{r}_b$	cm	Radius of body	
UFS	$\bar{V}_\infty$	cm/s	Freestream speed	
TFS	$\bar{T}_\infty$	K	Freestream temperature	
TWK	$\bar{T}_W$	K	Temperature of body surface	
ROFS	$\bar{\rho}_\infty$	gm/cm <sup>3</sup>	Freestream density	
U(1)	$u_{1w} = u_W/\sin\theta$	Dimensionless	Initial guess for (slip speed)/sin $\theta$	Figure 17
T(1)	$T_{1w}$	Dimensionless	Initial guess for nondimensional temperature of gas at outer edge of Knudsen layer	Figure 18
U(I)	$u_1$	Dimensionless	Initial guess for $u_1$ (see equation 18) at location I (I = 2, 3, ..., 50)	Figures 5-7
V(I)	$v_1$	Dimensionless	Initial guess for $v_1$ (see equation 19) at location I (I = 2, 3, ..., 50)	Figures 5-7
T(1)	$T_1$	Dimensionless	Initial guess for $T_1$ (see equation 20) at location I (I = 2, 3, ..., 50)	Figures 5-7
RO(I)	$\rho_1$	Dimensionless	Initial guess for $\rho_1$ (see equation 21) at location I (I = 2, 3, ..., 50)	Figures 5-7
P2(I)	$P_2$	Dimensionless	Initial guess for $P_2$ (see equation 22) at location I (I = 2, 3, ..., 50)	Figures 5-7
CO2(I)	$Y_{O_2}$	Dimensionless	Initial guess for mass fraction of O <sub>2</sub> at location I (I = 2, 3, ..., 50)	Figures 5-7

Computer Program Variable	Symbol Used in Report	Dimensions	Description	Suggested Source or Range of Values
CNOI(I)	$Y_{NO^+}$	Dimensionless	Initial guess for mass fraction of $NO^+$ at location I (I = 2, 3, ..., 50)	Figures 5-7
CNO(I)	$Y_{NO}$	Dimensionless	Initial guess for mass fraction of NO at location I (I = 2, 3, ..., 50)	Figures 5-7
CN(I)	$Y_N$	Dimensionless	Initial guess for mass fraction of N at location I (I = 2, 3, ..., 50)	Figures 5-7
CO(I)	$Y_O$	Dimensionless	Initial guess for mass fraction of O at location I (I = 2, 3, ..., 50)	Figures 5-7

The quantities U(I) through CO(I) must be input for each of the 49 equally-spaced points in the shock layer along the stagnation streamline starting with I = 2 near the wall and ending with I = 50 near the freestream (see Fig. A-1). The wall (I = 1) and freestream (I = 51) boundary conditions are given by the quantities UFS through T(1).

The format for the input data is as follows:

Card	Format	Variable Name(s)	Location and Description
1	12A6	AB1, ..., AB/2	Columns 1-72 contain a comment for project identification
2	3F10.0	XBOD	Columns 1-10 contain the body option (sphere = 1.0, cylinder = 2.0)
		XCAT	Columns 11-20 contain the wall catalyticity option (noncatalytic wall = 1.0, fully catalytic wall)
		XVIS	Columns 21-30 contain the viscosity option (Sutherland viscosity = 1.0, Hansen viscosity = 2.0)
3	2F10.0	CASE	Columns 1-10 contain the run number. This identification is useful to distinguish between runs whenever a rerun is necessary



Card	Format	Variable Name(s)	Location and Description
4	2F10.0	EFFR1	Columns 11-20
		TOL	Columns 1-10
		EPSI	Columns 11-20
5	5F10.0	ALT	Columns 1-10
		RADB	Columns 11-20
		UFS	Columns 21-30
		TFS	Columns 31-40
		TWK	Columns 41-50
6	E13.8	ROFS	Columns 1-13
7	2F10.0	U(1)	Columns 1-10
		T(1)	Columns 11-20
8-56	5E16.9	U(I)	Columns 1-16
		V(I)	Columns 17-32
		T(I)	Columns 33-48
		RO(I)	Columns 49-64
		P2(I)	Columns 65-80
57-105	5E16.9	CO2(I)	Columns 1-16
		CNOI(i)	Columns 17-32
		CNO(I)	Columns 33-48
		CN(I)	Columns 49-64
		CO(I)	Columns 65-80

## PROGRAM OUTPUT

The program printout is described in this section. Figure 5 of the text was obtained from this printout. Explanation of the printout follows.

### Title Page

Options used — The options which were selected for body, wall catalyticity, and viscosity are printed out here.

Input data — The input data are printed out for record.

Computed freestream data — Some freestream quantities which are used for nondimensionalizing and for computation of various similarity parameters are printed here. The equations used are as follows:

$$VSOUND = \sqrt{\gamma \bar{R} \bar{T}_{\infty}} \quad [\text{cm/sec}] \quad , \quad (\text{A-1})$$

where

$$\gamma = 1.4$$

$$\bar{R} = 2.8708 \times 10^6 \text{ cm}^2/\text{sec}^2\text{K} \quad .$$

$$M_{\infty} = AMACH = \frac{UFS}{VSOUND} \quad (\text{A-2})$$

$$\bar{T}_{0\infty} = STAGFS = \bar{T}_{\infty} \left( 1 + \frac{\gamma - 1}{2} M_{\infty}^2 \right) \quad [^{\circ}\text{K}] \quad (\text{A-3})$$

$$\bar{\mu}_{\infty} = RTFS = \frac{1.458 \times 10^{-5} \bar{T}_{\infty}}{\bar{T}_{\infty} + 110.4} \quad [\text{gm/cm}\cdot\text{sec}] \quad (\text{A-4})$$

$$\bar{\mu}_{0\infty} = RTSTAG = \frac{1.458 \times 10^{-5} \bar{T}_{0\infty}}{\bar{T}_{0\infty} + 110.4} \quad [\text{gm/cm}\cdot\text{sec}] . \quad (\text{A-5})$$

It is recognized that the stagnation temperature, equation (A-3), has little physical meaning at the high stagnation enthalpies for which this computer program is intended. This is because equation (A-3) is based on the assumption of constant specific heat for temperatures up to  $\bar{T}_{0\infty}$ ; the assumption of constant specific heat is violated at moderate temperatures. However,  $\bar{T}_{0\infty}$  is used only for nondimensionalizing and for computing  $\bar{\mu}_{0\infty}$ . It does not influence the computation of the flow field in any way. Likewise,  $\bar{\mu}_{0\infty}$  is a fictitious viscosity because  $\bar{T}_{0\infty}$  is fictitious and also because the Sutherland formula is not applicable at high temperatures.

**Similarity Parameters** — Some similarity parameters are printed out here. Several of these have been used, with some success, in correlating hypersonic or low density data. The similarity parameters are defined as follows:

$$\text{Re}_{\infty} = \text{REYF} = \frac{\bar{\rho}_{\infty} \bar{V}_{\infty} \bar{r}_b}{\bar{\mu}_{\infty}} , \quad (\text{A-6})$$

$$\text{Re}_{0\infty} = \text{REYN} = \frac{\bar{\rho}_{\infty} \bar{V}_{\infty} \bar{r}_b}{\bar{\mu}_{0\infty}} , \quad (\text{A-7})$$

$$\text{Re}_w = \text{REWALL} = \frac{\bar{\rho}_{\infty} \bar{V}_{\infty} \bar{r}_b}{\bar{\mu}_w} , \quad (\text{A-8})$$

where  $\bar{\mu}_w$  is computed by the Sutherland formula using  $\bar{T}_w$ .

$$\text{K}_{n_{\infty}} = \text{XKNFS} = \frac{\bar{\lambda}_{\infty}}{\bar{r}_b} , \quad (\text{A-9})$$

$$\bar{V} = \text{VBAR} = M_{\infty} \sqrt{\frac{C_{\infty}}{\text{Re}_{\infty}}} \quad , \quad (\text{A-10})$$

where

$$C_{\infty} = \frac{\bar{\mu}_w}{\bar{\mu}_{\infty}} \frac{\bar{T}_{\infty}}{\bar{T}_w} \quad .$$

$$K^2 = \text{XKSQ} = \frac{\text{Re}_{\infty}}{\gamma_{\infty} M_{\infty}^2 C_*} \quad , \quad (\text{A-11})$$

where

$$C_* = \frac{\bar{\mu}_*}{\bar{\mu}_{\infty}} \frac{\bar{T}_{\infty}}{\bar{T}_*}$$

and

$$\bar{T}_* = \frac{1}{2} (\bar{T}_{0\infty} + \bar{T}_w)$$

$$\bar{\mu}_* = \text{Sutherland viscosity computed at } \bar{T}_* \quad .$$

The similarity parameter  $\phi$  is given as the final item in the printout. It is placed in this location because it uses the computed enthalpy at the wall. This parameter was suggested by Potter (Rarefied Gas Dynamics, Supplement 5, Vol. I, 1969).

$$\phi = \text{PHI} = \text{Re}_w \left( \frac{2\bar{h}_w}{\bar{V}_{\infty}^2} \right)^{0.6} \quad . \quad (\text{A-12})$$

## Data Pages

A set of computed data is printed out after every 200 iterations. Each set of data consists of three pages.

1. First Page. The computed shock layer thickness (E<sub>FFR</sub>) and wall heat transfer coefficient ( $C_H$ ) are printed at the top of the page after every 50 iterations. The control factors which are used to update the computed quantities to obtain input variables for the next iteration are also printed here.

The columns on this page give nondimensional values of (1) radial position from the center of the body to the equally-spaced points at which the independent variables are computed, (2) the velocity component  $u_1$  [see equation (18)], (3) the velocity component  $v_1$  [see equation (19)], (4) specific enthalpy, (5) temperature, (6) viscosity ratio, local ratio of Hansen viscosity to Sutherland viscosity, (7) pressure  $P_1$  [see equation (22)], (8) pressure correction  $P_2$  [see equation (22)], and (9) density.

Note that VFS is the same as UFS elsewhere in the program. It is the freestream speed  $\bar{V}_\infty$ .

2. Second Page. This page gives the reaction rates for each of the seven species.

3. Third Page. This page gives the mass fractions of the chemical species which compose the air. Note that mass fraction was denoted by "Y" in the text rather than by "C." The electron number density is also given.

When  $C_H$  converges within the prescribed tolerance, or when the iterations reach 2000, the iterations are stopped and the final data set is printed out. At this point a set of 98 cards is punched to record the velocity and thermodynamic data given on page 1 and the mass fraction data on page 3 of the data printout. This set of cards can then be used as starting data for a new run.

To determine whether or not the program has converged sufficiently and whether the results are reasonable, the following checks are suggested.

- (1) Check E<sub>FFR</sub> for convergence
- (2) Check  $C_H$  for convergence

(3) Check the slip conditions for convergence, i.e. compare  $U_1$  and  $T$  at  $RAD/RADB = 1.0$  for the last few sets of data

(4) Scan the velocity component and thermodynamic property columns to determine if the profiles are smooth and the values are reasonable

(5) Scan the mass fraction columns to check for reasonable behavior. Particularly check the  $N_2$  column near the wall to determine if these mass fractions exceed the freestream value.

## SOME DIFFICULTIES ENCOUNTERED IN RUNNING PROGRAM

The program generally runs without much difficulty if the starting data are within reasonable bounds; however, problems do occur, and some of the most common problems are discussed as follows.

1. Failure of EFR to Converge. The shock layer thickness, EFR, usually is the first quantity to converge; however, it sometimes diverges. The most common cause of this is using starting data from a run with a much different stagnation enthalpy, or  $\bar{T}_{0\infty}$ , from the case being run. This causes a discontinuity in the nondimensional temperature profile between the points  $I = 50$  and  $I = 51$  because the point  $I = 50$  comes from the starting data from a previous run and point  $I = 51$  is the freestream boundary condition. The decision on whether to decrease, increase, or not change EFR is made on the basis of the temperature gradient near the freestream (see Appendix B). Therefore, if a sizable discontinuity in temperature exists at this location, EFR will diverge.

One solution to this problem is to modify the last few input cards to make a smooth transition in the temperature profile to the freestream temperature. Probably a better solution would be to change the nondimensionalizing temperature from  $\bar{T}_{0\infty}$  to  $\bar{T}_{\infty}$ . This would guarantee a smooth transition, in the starting data, to the freestream temperature. To prevent divergence of EFR, the program has been modified to restrict EFR within the range

$$\frac{1}{2} \text{ EFFR1} \leq \text{EFFR} \leq \frac{3}{2} \text{ EFFR1} .$$

If EFFR1 is not guessed closely enough, this restriction will necessitate a rerun using a different EFFR1.

2. Unrealistic Chemical Composition. If the species mass fractions in the starting data are greatly different from the true values for the case being run, unrealistic computed mass fractions can result. This can lead to unrealistic computed values for all other quantities. For example, this might occur for a rather low speed, high density case in which dissociation should be negligible (Fig. 11). However, since starting data from a low speed, high density run were not available, data were used from a high speed, high density run in which the degree of dissociation was high. The program could not handle the grossly incorrect starting data, and the computed results became more and more unrealistic from one iteration to the next. The unrealistic data included mass fractions for  $\text{N}_2$  much larger than the freestream value, negative  $C_H$  (heat transferred from the body to the gas) and a monotonically increasing EFFR beyond reasonable bounds (Fig. 16). Note that  $C_H$  was called "STANC" in the older printout.

## SOME SUGGESTIONS FOR RUNNING PROGRAM

To minimize the previously discussed problems and to facilitate operation of the program, the following suggestions on input data are made.

1. EFFR1, U(1), T(1), and TOL. Figures 16, 17, and 18 can be used for obtaining initial guesses for EFFR1, U(1), and T(1), respectively. The value for TOL probably should be set smaller than the desired tolerance on  $C_H$  to assure that the other quantities converge also. A value of 0.00001 for TOL was used in most of the runs made for this report.

2. Velocity, Thermodynamic Property, and Mass Fraction Profiles. For best results, the starting data profiles for the quantities U(I) through CO(I) should be obtained from a run which matches the required freestream conditions of the new run as closely as possible. Close matching is especially important at low altitude, high density conditions for which gradients in the flow properties are large. The question arises as to what constitutes a

sufficiently close "match" and which matching, or similarity, parameter should be used. If every freestream and body condition is matched fairly closely, it is unnecessary to use a similarity parameter. From experience, it is found that, for a given body, changes in freestream density,  $\bar{\rho}_{\infty}$ , and speed,  $\bar{V}_{\infty}$ , in that order, have the greatest effect on the outcome of the run. Therefore, for bodies of the same size, one should try to match  $\bar{\rho}_{\infty}$  and  $\bar{V}_{\infty}$ . A good rule of thumb is that  $\bar{\rho}_{\infty}$  for the new run should not differ from  $\bar{\rho}_{\infty}$  for the starting data run by more than the equivalent of 3 km altitude in the standard atmosphere. Near the low altitude extreme of the program's capability, this difference should not be more than approximately 1 km. A difference in  $\bar{V}_{\infty}$  of approximately 2000 m/sec can usually be tolerated. For bodies of greatly different size, one should probably use a similarity parameter, such as  $K^2$ , to match starting data to the new run.

Figures 5, 6, and 7 can be used for starting data. If the desired freestream properties for the new run are farther removed from those of Figures 5, 6, and 7 than the previously suggested increments, it is advisable to reach the desired conditions in two or more runs. If severe difficulties like those previously discussed are encountered, it is probably best to go back to a "good," or converged, set of starting data and use smaller increments in freestream properties rather than to use the output of the "bad" run as starting data for a rerun. If the program is used frequently, a library of starting data card sets can be built up to cover the range of possible freestream conditions for which the program is applicable.

3. Hansen-Sutherland Viscosity Option. If the computed ratio of Hansen's viscosity to Sutherland's viscosity is approximately 1.0 for a given set of freestream conditions, one can use the Sutherland viscosity option for nearby freestream conditions to reduce computer time.



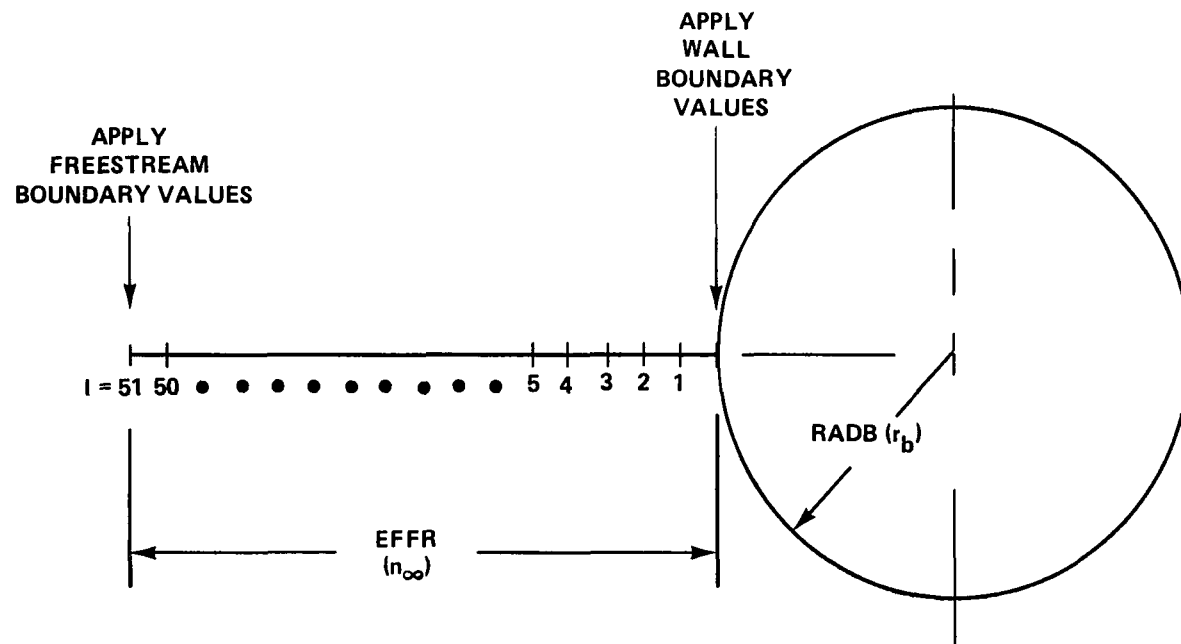


Figure A-1. Location of computation points.



## APPENDIX B. LISTING OF PROGRAM

MAIN

DATE 120877

```

00100 1* C SUCCESSIVE ACCELERATED REPLACEMENT METHOD 000000
00100 2* C NONEQUILIBRIUM FLOW IN THE MERGED SHOCK LAYER OF A BLUNT BODY 000000
00100 3* C NEAR THE STAGNATION REGION USING FULL N.S. EQUATIONS 000000
00100 4* C SLIP CONDITIONS AT THE WALL 000000
00100 5* C CATALYTIC OR NONCATALYTIC WALL 000000
00100 6* C ***** 000000
00100 7* C 000000
00101 8* C DIMENSION U(100),V(100),T(100),H(100),P(100),P2(100),RT(100), 000000
00101 9* C 1D(100),RAD(100),X(100),C02(100),CNZ(100),CNO(100),CN(100),C0(100), 000000
00101 10* C 1CNOI(100),CEL(100),RO(100) 000000
00103 11* C DIMENSION W02(100),WN2(100),WNO(100),WN(100),W0(100),WNOI(100), 000001
00103 12* C 1WEL(100),PCL(100) 000001
00104 13* C DIMENSION HN2(100),H02(100),H0(100),HN(100),HNO(100),HNOI(100) 000001
00105 14* C DIMENSION HCL(100) 000001
00106 15* C DIMENSION C1(10),C2(10),C3(10),PP1(10),W(10),EL(10) 000001
00106 16* C 000001
00106 17* C INPUT HANSEN'S VISCOSITY DATA 000001
00107 18* C DIMENSION XMUSU(100),TAB5(100),XMURAT(100),XMU(100) 000001
00110 19* C DIMENSION TEMP(37),TAB1(37),TAB2(37),TAB3(37),TAB4(37) 000001
00111 20* C DATA TEMP /2000.0,2500.0,3000.0,3500.0,4000.0,4500.0,5000.0, 000001
00111 21* C 1 5500.0,6000.0,6500.0,7000.0,7500.0,8000.0,8500.0,9000.0,9500.0, 300001
00111 22* C 2 10000.0,10500.0,11000.0,11500.0,12000.0,12500.0,13000.0,13500.0, 000001
00111 23* C 3 14000.0,14500.0,15000.0,15500.0,16000.0,16500.0,17000.0,17500.0, 000001
00111 24* C 4 18000.0,18500.0,19000.0,19500.0,20000.0/ 000001
00113 25* C DATA TAB1 /C.886,C.846,C.830,C.815,C.803,C.792,C.782,C.773,C.764, 000001
00113 26* C 1 0.757,C.750,C.743,C.737,C.731,C.725,C.720,C.715,C.710,C.706, 000001
00113 27* C 2 0.701,C.697,C.693,C.689,C.685,C.681,C.677,C.672,C.668,C.664, 000001
00113 28* C 3 0.660,C.656,C.651,C.647,C.643,C.639,C.635,C.630/ 000001
00115 29* C DATA TAB2 /C.742,C.705,C.676,C.650,C.628,C.608,C.591,C.575,C.561, 000001
00115 30* C 1 0.548,C.536,C.524,C.514,C.504,C.495,C.486,C.478,C.470,C.463, 000001
00115 31* C 2 0.456,C.448,C.443,C.437,C.431,C.426,C.420,C.415,C.407,C.401, 000001
00115 32* C 3 0.395,C.389,C.384,C.376,C.371,C.365,C.359,C.352/ 000001
00117 33* C DATA TAB3 /C.468,C.457,C.445,C.434,C.423,C.412,C.401,C.397,C.380, 000001
00117 34* C 1 0.366,C.353,C.342,C.331,C.321,C.313,C.304,C.297,C.290,C.283, 000001
00117 35* C 2 0.281,C.270,C.266,C.261,C.256,C.252,C.247,C.243,C.236,C.230, 000001
00117 36* C 3 0.224,C.218,C.213,C.206,C.201,C.196,C.190,C.185/ 000001
00121 37* C DATA TAB4 /123.30,122.20,120.80,118.10,114.80,109.70,120.00, 000001
00121 38* C 1 89.90,75.60,64.50,55.70,48.60,42.80,37.90,33.80,30.40,27.40, 000001
00121 39* C 2 24.90,22.70,20.80,19.09,17.60,16.27,15.10,14.04,13.09,12.24, 000001
00121 40* C 3 11.25,10.98,10.48,10.01,9.50,9.01,8.75,8.01,7.51,7.68/ 000001
00121 41* C ***** 000001
00121 42* C 000001
00121 43* C 000001
00123 44* C READ(5,702) AB1, AB2, AB3, AB4, AB5, AB6, AB7, AB8, AB9, AB10, AB11, AB12 000001
00141 45* 702 FORMAT(12A6) 000022
00142 46* READ(5,700) XB0D,XCAT,XVIS 000022
00147 47* READ(5,700) CASE,EFFRI 000032
00153 48* READ(5,700) TOL,EPSI 000041
00157 49* READ(5,700) ALT,RADB,UFS,TFS,TWK 000050
00166 50* 700 FORMAT(6F10.0) 000062
00167 51* READ(5,701)IROFS 000062
00172 52* 701 FORMAT(13.8) 000070
00173 53* EFFR=EFFRI 000070
00174 54* ICASE=CASE 000072
00174 55* C 000072

```

MAIN				DATE 120877
00174	56*	C	*****	000072
00174	57*	C		000072
00174	58*	C	BOUNDARY CONDITIONS AT THE BODY	000072
00175	59*		READ(5,700)U(1),T(1)	000101
00201	60*		V(1)=0.	000110
00201	61*	C	*****	000110
00201	62*	C		000110
00201	63*	C	EL REPRESENTS LEWIS NUMBER AND A IS AVAGADRO NUMBER	000110
00202	64*		DO 1 K=1,7	000114
00205	65*	1	EL(K)=1.4	000114
00207	66*		EL1=1.4	000116
00210	67*		EL2=1.4	000117
00211	68*		PR=0.75	000120
00212	69*		GAMMA=1.4	000122
00213	70*		SC1=PR/EL1	000123
00214	71*		SC2=PR/EL2	000125
00214	72*	C		000125
00214	73*	C	*****	000125
00214	74*	C		000125
00214	75*	C	PRINT PROGRAM TITLE AND INPUT DATA	000125
00215	76*		PRINT 10	000130
00217	77*	10	FORMAT(1H1,///25X,'HYPERSONIC, MERGED SHOCK LAYER, STAGNATION LINE	000134
00217	78*	1	FLOW FIELD FOR SPHERE OR CIRCULAR CYLINDER')	000134
00220	79*		PRINT 11	000134
00222	80*	11	FORMAT(/50X,'JAIN-KUMAR-HENDRICKS PROGRAM')	000140
00223	81*		PRINT 12	000140
00225	82*	12	FORMAT(45X,'(OBTAINED FROM BILL HENDRICKS, JULY 1975)')	000144
00226	83*		PRINT 13	000144
00230	84*	13	FORMAT(///40X,'FEATURES: SIMILARITY SOLUTION OF NAVIER-STOKES EQU	000150
00230	85*		ATIONS')	000150
00231	86*		PRINT 14	000150
00233	87*	14	FORMAT(50X,'NONEQUILIBRIUM CHEMISTRY'	000154
00233	88*	1	/50X,'SLIP BOUNDARY CONDITIONS')	000154
00234	89*		PRINT 19	000154
00236	90*	19	FORMAT(//31X,'AVAILABLE OPTIONS:')	000160
00237	91*		PRINT 21	000160
00241	92*	21	FORMAT(50X,35H1. BODY: (A) SPHERE OR (B) CYLINDER)	000164
00242	93*		PRINT 22	000164
00244	94*	22	FORMAT(50X, '2. WALL CATALYTICITY: (A) NONCATALYTIC WALL OR (B) F	000170
00244	95*		ULLY CATALYTIC WALL')	000170
00245	96*		PRINT 23	000170
00247	97*	23	FORMAT(50X, '3. VISCOSITY: (A) SUTHERLAND OR (B) HANSEN'S HIGH TE	000174
00247	98*		MP. MODEL (INACA TR R-50)')	000174
00250	99*		PRINT 24	000174
00252	100*	24	FORMAT(/35X,'MODIFICATIONS:')	000200
00253	101*		PRINT 25	000200
00255	102*	25	FORMAT(50X, '1. 9/2/76 (KEN JOHNSTON) ENTHALPY COMPUTATION MODIFIED	000204
00255	103*	1	TO ALLOW FOR PARTIALLY 75X, 'EXCITED VIBRATIONAL STATE (VINCENTI	000204
00255	104*	2	AND KRUGER, P.135) STATEMENTS 30,130. /53X, 'PREVIOUS ASSUMPTION	000204
00255	105*	3:	FULLY EXCITED VIBRATIONAL STATE')	000204
00256	106*		PRINT 26	000204
00260	107*	26	FORMAT(750X, '2. 9/2/76 (KEN JOHNSTON) OPTION ADDED TO COMPUTE VISC	000210
00260	108*		OSITY BY HANSEN'S HIGH TEMP. /55X, 'MODEL. (SUTHERLAND'S FORMULA	000210
00260	109*	2	BECOMES INACCURATE AT HIGH TEMP.')	000210
00261	110*		PRINT 350	000210
00263	111*	350	FORMAT(/50X, '3. 9/9/76 (KEN JOHNSTON) CATALYTIC AND NONCATALYTIC P	000214

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00263	112*	1 PROGRAMS COMBINED.'/55X,'(OPTION AVAILABLE TO CHOOSE DESIRED WALL C	000214
00263	113*	ZATALYTIC CONDITION)'))	000214
00264	114*	PRINT 351	000214
00266	115*	351 FORMAT(/50X,'4. 9/9/76 (KEN JOHNSTON) SPHERE AND CYLINDER PROGRAMS	000220
00266	116*	1 COMBINED.'/55X,'(OPTION AVAILABLE TO CHOOSE DESIRED BODY)'))	000220
00267	117*	PRINT 18,AB1,AB2,AC3,AB4,AB5,AB6,AB7,AB8,AB9,AB10,AB11,AB12	000220
00305	118*	18 FORMAT(/5X,'PROJECT: ',12A6)	000240
00306	119*	PRINT 27	000240
00310	120*	27 FORMAT(/13X,'OPTIONS USED:')	000244
00311	121*	IF(XBOD-0.9 .GT. 0.2) GO TO 29	000244
00313	122*	PRINT 28	000251
00315	123*	28 FORMAT(27X,'SPHERE')	000255
00316	124*	GO TO 41	000255
00317	125*	29 PRINT 40	000257
00321	126*	40 FORMAT(27X,'CIRCULAR CYLINDER')	000263
00322	127*	41 IF(XCAT-0.9 .GT. 0.2) GO TO 43	000263
00324	128*	PRINT 42	000267
00326	129*	42 FORMAT(27X,'NONCATALYTIC WALL')	000273
00327	130*	GO TO 45	000273
00330	131*	43 PRINT 44	000275
00332	132*	44 FORMAT(27X,'FULLY CATALYTIC WALL')	000301
00333	133*	45 IF(XVIS-0.9 .GT. 0.9) GO TO 47	000301
00335	134*	PRINT 46	000305
00337	135*	46 FORMAT(27X,'SUTHERLAND VISCOSITY FORMULA')	000311
00340	136*	GO TO 49	000311
00341	137*	47 PRINT 48	000313
00343	139*	48 FORMAT(27X,'HANSEN'S HIGH TEMP VISCOSITY FORMULA')	000317
00344	139*	49 PRINT 15,ICASE,EFFRI,U(1),T(1),TOL,EPSI	000317
00354	140*	15 FORMAT(/19X,'RUN NO.: ',I4,	000330
00354	141*	1 //15X,'INPUT DATA:')	000330
00354	142*	1 //18X,'EFFECTIVE RADIUS (EFFRI) = ',F7.4,' (STANDOFF DISTANCE/BO	000330
00354	143*	1DY RADIUS)')	000330
00354	144*	2 /36X,'U1(1) = ',F9.5,' (U1BAR(1)/FREESTREAM SPEED)'	000330
00354	145*	3 /36X,'T1(1) = ',F8.5,' (T1BAR(1)/FREESTREAM STAG. TEMP.)'	000330
00354	145*	4 /4X,'HEAT TRANSFER COEFF. TOLERANCE (TOL) = ',F8.6,	000330
00354	147*	5 /27X,'EPSI = ',F8.6,')	000330
00355	149*	PRINT 16,ALT,RADB,UFS,TFS	000330
00363	149*	16 FORMAT(27X,17HALTITUDE (ALT) = ',F9.4,' 4H KM/23X,21HBODY RADIUS (R	000340
00363	150*	1ADB) = ',F9.4,' 4H CM/19X,25HFREESTREAM SPEED (UFS) = ',F11.3,' 8H C	000340
00363	151*	2M/SEC/13X,31HFREESTREAM TEMPERATURE (TFS) = ',F9.4,'12H DEG KELVIN)	000340
00364	152*	PRINT 17,ROFS,TWK	000340
00370	153*	17 FORMAT(16X,'FREESTREAM DENSITY (ROFS) = ',E10.5,' GM/CM**3'	000346
00370	154*	1 /19X,'WALL TEMPERATURE (TWK) = ',F9.4,' DEG KELVIN')	000346
00370	155*	C	000346
00370	156*	C	000346
00370	157*	C	000346
00370	158*	C	000346
00371	159*	COMPUTE AND PRINT FREESTREAM CONDITIONS AND SIMILARITY PARAMETERS	000346
00372	160*	VSCOND=100.0*(1.4*287.08*TFS)**0.5	000346
00373	161*	RTFS=(1.458E-05*TFS**1.5)/(TFS+110.4)	000357
00374	162*	AMACH=UFS/VSCOND	000371
00375	163*	STAGFS=TFS*(1.0+((GAMMA-1.0)/2.0)*AMACH**2)	000374
00376	164*	RTSTAG=(1.458E-05*STAGFS**1.5)/(STAGFS+110.4)	000404
00377	165*	RTWALL=(1.458E-05*TWK**1.5)/(TWK+110.4)	000416
00400	166*	REYF=UFS*ROFS*RADB/RTFS	000430
00401	167*	REYN=REYF*RTFS/RTSTAG	000435
		REWALL=REYF*RTFS/RTWALL	000441

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00402	163*	XLAMDA=(1.2562*RTFS)/(R0FS*(2.8702E+06*TF5)**0.5)	000444
00403	169*	XKNFS=XLAMDA/RAD3	000460
00404	173*	CFGS=RTWALL*TF5/(RTFS*TWK)	000462
00405	171*	VBAR=AMACH*(CFGS/REYF)**0.5	000470
00406	172*	TSTAR=C.5*(STAGFS*TWK)	000500
00407	172*	RTSTAR=(1.458E-05*TSTAR**1.5)/(TSTAR+110.4)	000504
00410	174*	CSTAR=RTSTAR*TF5/(RTFS*TSTAR)	000515
00411	175*	XKSG=REYF/(CAMMA*AMACH**2*CSTAR)	000522
00412	176*	PRINT 400	000530
00414	177*	400 FORMAT(// 'COMPUTED FREESTREAM DATA: '//)	000534
00415	179*	PRINT 401,VSCUND,AMACH,STAGFS	000534
00422	179*	401 FORMAT(7X,'FREESTREAM SPEED OF SOUND (VSCUND) = ',F10.3,' CM/SEC'	000543
00422	130*	1 /11X,'FREESTREAM MACH NUMBER (AMACH) = ',F6.3/12X,'FREESTREAM S	000543
00422	181*	2TAC TEMP (STAGFS) = ',F9.3,' DEG KELVIN')	000543
00423	132*	PRINT 402,RTFS,RTSTAC	000543
00427	137*	402 FORMAT(14X,'FREESTREAM VISCOSITY (RTFS) = ',E10.5,' CM/CM-SEC'	000551
00427	134*	1 /7X,'FREESTREAM STAC VISCOSITY (RTSTAG) = ',E10.5,' GH/CM-SEC')	000551
00420	185*	PRINT 403	000551
00432	196*	403 FORMAT(4X,'SIMILARITY PARAMETERS: '//)	000555
00433	187*	PRINT 404,REYF,REYN,REWALL,XKNFS,VGAR,XKSG	000555
00443	133*	404 FORMAT(3X,'FREESTREAM REYNOLDS NUMBER (REYF) = ',F9.2,	000567
00443	189*	1 /8X,'STAGNATION REYNOLDS NUMBER (REYN) = ',F9.2,	000567
00443	190*	2 /12X,'WALL REYNOLDS NUMBER (REWALL) = ',F9.2,	000567
00443	191*	3 /8X,'FREESTREAM KNUDSEN NUMBER (XKNFS) = ',F9.4,	000567
00443	192*	4 /37X,'VGAR = ',F9.4,	000567
00443	193*	5 /25X,'K SQUARED (XKSG) = ',F9.4, '//)	000567
00443	194*	C	000567
00443	195*	C NOTE THE SUBSTITUTION REYN=REYF FOR REMAINDER OF PROGRAM	000567
00444	196*	REYN=REYF	000567
00445	197*	PRINT 180	000571
00445	198*	C	000571
00445	199*	C *****	000571
00445	200*	C	000571
00447	201*	C UNLR=1.087	000575
00450	202*	A=6.0222*10.**23	000577
00451	207*	WTN2=28.0	000601
00452	204*	WT02=32.0	000603
00453	205*	WTN=14.0	000605
00454	206*	WT0=16.0	000607
00455	207*	WTN0=30.0	000611
00456	208*	WTN0I=30.0	000613
00457	209*	WTCL=1./1820.	000614
00460	210*	NDIV=50	000616
00461	211*	DIVN=NDIV	000620
00462	212*	DELTA=1./DIVN	000623
00463	213*	NDIV1=NDIV+1	000626
00464	214*	HALT=25.0	000633
00464	215*	C	000633
00464	216*	C *****	000633
00464	217*	C	000633
00465	218*	READ 300,(U(I),V(I),T(I),RO(I),P2(I),I=2,NDIV)	000635
00477	219*	READ 300,(C02(I),CNOI(I),CNOI(I),CN(I),C0(I),I=2,NDIV)	000653
00477	220*	C	000653
00477	221*	C *****	000653
00477	222*	C	000653
00477	223*	C BOUNDARY CONDITIONS AT FREESTREAM	000653

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00511	224*	U(NDIV1)=1.	000670
00512	225*	V(NDIV1)=-1.	000672
00513	226*	T(NDIV1)=TFS/STAGFS	000673
00514	227*	RG(NDIV1)=1.	000676
00515	228*	RT(NDIV1)=1.0	000677
00516	229*	P2(NDIV1)=0.	000700
00517	230*	CN2(NDIV1)=0.767	000701
00520	231*	CO2(NDIV1)=0.233	000703
00521	232*	CNO(NDIV1)=0.	000705
00522	233*	CO(NDIV1)=0.	000706
00523	234*	CN(NDIV1)=0.	000707
00524	235*	CNOI(NDIV1)=0.	000710
00525	235*	CEL(NDIV1)=0.	000711
00525	237*	C *****	000711
00525	233*	C *****	000711
00525	239*	C *****	000711
00525	240*	C CATALYTIC BOUNDARY CONDITIONS	000711
00525	241*	C *****	000711
00525	242*	C *****	000711
00526	247*	C IF(XCAT-C.9 .LT. 0.2)GO TO 450	000711
00530	244*	C CN2(1)=0.767	000712
00531	245*	C CO2(1)=0.233	000717
00532	246*	C CNO(1)=0.	000720
00533	247*	C CN(1)=0.0	000721
00534	248*	C CO(1)=0.0	000722
00535	249*	C CNOI(1)=CNOI(2)	000723
00536	250*	C CEL(1)=CNOI(2)*WTEL/WTNOI	000724
00537	251*	C PEL(1)=A*RO(1)*ROFS*CNOI(1)/WTNOI	000726
00540	252*	C GO TO 451	000732
00540	253*	C *****	000740
00540	254*	C *****	000740
00540	255*	C NONCATALYTIC WALL BOUNDARY CONDITIONS (FOR FIRST ITERATION)	000740
00540	256*	C *****	000740
00540	257*	C *****	000740
00541	253*	C 450 CN2(2)=1.0-CO2(2)-CNO(2)-CN(2)-CO(2)-CNOI(2)*(1.0*WTEL/WTNOI)	000742
00542	259*	C CN2(1)=CN2(2)	000754
00543	260*	C CO2(1)=CO2(2)	000755
00544	261*	C CNO(1)=CNO(2)	000757
00545	262*	C CN(1)=CN(2)	000761
00546	263*	C CO(1)=CO(2)	000763
00547	264*	C CNOI(1)=CNOI(2)	000765
00550	265*	C CEL(1)=CNOI(2)*WTEL/WTNOI	000767
00551	266*	C PEL(1)=0.	000773
00551	267*	C *****	000773
00551	268*	C *****	000773
00551	269*	C *****	000773
00551	270*	C INITIAL GUESS FOR CNOI,CEL AND H	000773
00552	271*	C 451 HIN2=0.0	000775
00553	272*	C HIO2=0.	000775
00554	273*	C HINO=3050.*10.**7	000776
00555	274*	C HIN=33620.*10.**7	001000
00556	275*	C HIO=15430.*10.**7	001002
00557	276*	C HINOI=9000.*10.**7	001004
00560	277*	C HICL=0.	001006
00561	278*	C W1=8.3143*10.**7*STAGFS/UFS*.2	001007
00562	279*	C DO 30 I=1,NDIV1	001200



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00565	280.	CN2(I)=1.-C02(I)-CNO(I)-CN(I)-C0I(I)-CNOI(I)*(1.+WTCL/WTNOI)	001216
00566	281.	CCL(I)=CNOI(I)*WTCL/WTNOI	001226
00567	282.	W31=T(I)*STAGFS	001232
00570	283.	VN2=3390./W31	001235
00571	284.	V02=2270./W31	001240
00572	285.	VNO=2740./W31	001243
00573	286.	VNOI=2740./W31	001246
00574	287.	IF(VN2 .GT. 80.C)GO TO 455	001247
00576	289.	VID=(CN2(I)/28.)*VN2/(EXP(VN2)-1.)+(C02(I)/32.)*V02/(EXP(V02)-1.)	001252
00576	289.	1+(CNO(I)/30.)*VNO/(EXP(VNO)-1.)+(CNOI(I)/30.)*VNOI/(EXP(VNOI)-1.)	001252
00577	290.	GO TO 456	001317
00600	291.	455 VIS=0.C	001321
00601	292.	456 CONTINUE	001322
00602	293.	H(I)=3.5*(CN2(I)/28.+C02(I)/32.+CNO(I)/30.+CNOI(I)/30.)*VIS	001322
00602	294.	1+2.5*(CN(I)/14.+C0(I)/16.+CNOI(I)/30.)	001322
00603	295.	3C H(I)=W1*T(I)*H(I)+(CNO(I)*HINO+CN(I)*HIN+C0(I)*HIO+CNOI(I)*HINCI)	001347
00603	295.	1/UTS**2	001347
00603	297.	C	001347
00603	298.	C *****	001347
00603	299.	C	001347
00605	300.	DO 500 K=1,2000	001374
00610	301.	Q=EFFR*CLTA	001377
00610	302.	C TO FIND P AND MUE AT ALL POINTS	001377
00611	303.	DO 50 I=1,NCIV1	001406
00614	304.	TABS(I)=T(I)*STAGFS	001406
00615	305.	IF(I .EQ. 1) GO TO 31	001411
00617	307.	P(I)=RO(I)*T(I)*W1*(CN2(I)/WTN2+C02(I)/WT02+CNO(I)/WTNO+CN(I)/WTN	001414
00620	308.	1+C0(I)/WTO+2.*CNOI(I)/WTNOI)	001414
00621	309.	31 XMUSU(I) = 1.458E-05*TABS(I)**1.5/(TABS(I) + 110.4)	001444
00621	309.	IF(XVIS-C.9 .LT. 0.2)GO TO 33	001457
00623	310.	IF(TABS(I) .GE. 2000.0) GO TO 32	001461
00625	311.	33 XMURAT(I)=1.C	001466
00626	312.	GO TO 37	001467
00627	313.	32 TABS=TABS(I)	001471
00630	314.	CALL INTERP(1,4,1,37,TEMP,TABS)	001472
00631	315.	CALL INTERP(2,4,1,37,TAB1,SN2N)	001502
00632	316.	CALL INTERP(2,4,1,37,TAB2,SNN)	001512
00633	317.	CALL INTERP(2,4,1,37,TAB3,SNE)	001522
00634	318.	CALL INTERP(2,4,1,37,TAB4,SEE)	001532
00635	319.	XLPM2=1.03329*CN2(I)+.87542*C02(I)+2.5310*CN(I)*SN2N	001542
00635	320.	1 +2.12036*C0(I)*SN2N+.94819*CNO(I)+.94819*CNOI(I)	001542
00636	321.	XLMO2=1.0696*CN2(I)+.90413*C02(I)+2.6488*CN(I)*SN2N	001566
00636	322.	1 +2.2146*C0(I)*SN2N+.9803*CNO(I)+.9803*CNOI(I)	001566
00637	323.	XLMN=.8949*CN2(I)*SN2N+.7665*C02(I)*SN2N+2.0666*CN(I)*SNN	001612
00637	324.	1 +1.7508*C0(I)*SNN+.8259*CNO(I)+.8259*CNOI(I)*SN2N	001612
00640	325.	XLMO=.9159*CN2(I)*SN2N+.7830*C02(I)*SN2N+2.1391*CN(I)*SNN	001642
00640	326.	1 +1.8083*C0(I)*SNN+.8444*CNO(I)*SN2N+.8444*CNOI(I)*SN2N	001642
00641	327.	XLMO=1.0516*CN2(I)+.9899*C02(I)+2.5906*CN(I)*SN2N	001672
00641	328.	1 +2.1680*C0(I)*SN2N+.9644*CNO(I)+.9644*CNOI(I)	001672
00642	329.	XLMOI=1.0516*CN2(I)+.8759*C02(I)+3.2475*CN(I)*SN2N	001716
00642	330.	1 +2.1680*C0(I)*SN2N+.9644*CNOI(I)+.9644*CNOI(I)	001731
00643	331.	XLMOI=.7307*CN2(I)*SNN+.6393*C02(I)*SNN+1.4613*CN(I)*SNE	001731
00643	332.	1 +.2786*C0(I)*SNE+.6819*CNO(I)*SNN+.6819*CNOI(I)*SEE	001731
00643	333.	2 +52656.24*CCL(I)*SEC	001765
00644	334.	XMURAT(I)=1.0165*CN2(I)/XLMO2+.9509*C02(I)/XLMO2	001765
00644	335.	1 +1.4376*CN(I)/XLMO+1.8083*C0(I)/XLMO+.9820*CNO(I)/XLMO	001765

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00644	335*	2 +D.9820*CN0I(I)/XLMNOI + 229.469*CEL(I)/XLMEL	001765
00645	337*	37 XMU(I) = XMURAT(I)*XMUSU(I)	002022
00646	333*	50 RT(I) = XMU(I)/RTFS	002024
00646	339*	C TO FIND PRESSURE ON THE SURFACE USING MOMENTUM EQUATION	002024
00650	340*	U10=(4.*U(2)-U(3)-3.*U(1))/DELTA/2.	002030
00651	341*	V10=(4.*V(2)-V(3)-3.*V(1))/DELTA/2.	002041
00652	342*	RT10=(4.*RT(2)-RT(3)-3.*RT(1))/DELTA/2.	002051
00653	343*	V20=-V(4)+4.*V(3)-5.*V(2)+2.*V(1)	002062
00654	344*	DCL4V=V(5)-4.*V(4)+6.*V(3)-4.*V(2)+V(1)	002074
00655	345*	V20=V20+11./12.*DCL4V	002105
00656	346*	V20=V20/DCLTA**2	002110
00656	347*	C SPHERE/CYLINDER OPTION FOLLOWS	002110
00657	348*	IF(XSOD-0.9 .GT. 0.2) GO TO 51	002112
00657	349*	C SPHERE	002112
00661	350*	P10=V20*RT(1)/CFFR+V10*(2.*RT(1)+RT10/CFFR)+U10*RT(1)/2.-U(1)*(	002114
00661	351*	IRT10+3.5*RT(1)*CFFR)	002114
00662	352*	GO TO 52	002137
00662	353*	C CYLINDER	002137
00663	354*	51 P10=V20*RT(1)/CFFR+V10*(RT(1)+RT10/CFFR)+U10*RT(1)/4.-U(1)*	002141
00663	355*	1 (PT10/2.-C/4.-C*RT(1)*CFFR)	002141
00664	356*	52 P10=P10*.C/3.-C/RCYN	002166
00665	357*	P(1)=P(2)-DELTA*P10	002172
00666	358*	RO(1)=P(1)/W1/T(1)/(CN2(1)/WTN2+C02(1)/WT02+CN0(1)/WTN0+CN(1)/WTN	002175
00666	359*	1+CO(1)/WT0+2.*CN0I(1)/WTN0I)	002175
00667	360*	DO 60 N=2,NDIV	002252
00672	361*	EN=N	002267
00673	362*	SC X(N)=1.+(CN-1.)*Q	002272
00673	363*	C	002272
00673	364*	C SLIP CONDITIONS	002272
00673	365*	C	002272
00675	366*	AC=1.C	002301
00676	367*	W(1)=WTN2	002303
00677	368*	W(2)=WT02	002305
00700	369*	W(3)=WTN	002307
00701	370*	W(4)=WT0	002311
00702	371*	W(5)=WTN0	002315
00703	372*	W(6)=WTN0I	002315
00704	373*	W(7)=WTEL	002317
00705	374*	C1(1)=CN2(1)	002321
00706	375*	C1(2)=C02(1)	002323
00707	376*	C1(3)=CN(1)	002325
00710	377*	C1(4)=C0(1)	002327
00711	378*	C1(5)=CN0(1)	002331
00712	379*	C1(6)=CN0I(1)	002333
00713	380*	C1(7)=CEL(1)	002335
00714	381*	C2(1)=CN2(2)	002337
00715	382*	C2(2)=C02(2)	002341
00716	383*	C2(3)=CN(2)	002343
00717	384*	C2(4)=C0(2)	002345
00720	385*	C2(5)=CN0(2)	002347
00721	386*	C2(6)=CN0I(2)	002351
00722	387*	C2(7)=CEL(2)	002353
00723	388*	C3(1)=CN2(3)	002355
00724	389*	C3(2)=C02(3)	002357
00725	390*	C3(3)=CN(3)	002361
00726	391*	C3(4)=C0(3)	002363

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00727	337*	C3(5)=CNO(3)	002365
00730	393*	C3(6)=CNOI(3)	002367
00731	394*	C3(7)=CEL(3)	002371
00732	395*	TW=TWK/STAGFS	002373
00733	396*	SUM1=0.0	002375
00734	397*	DO 61 J=1,7	002406
00737	398*	61 SUM1=SUM1+C1(J)/W(J)	002406
00741	399*	LO 62 J=1,7	002415
00744	400*	62 PP1(J)=(C1(J)/W(J))*P(1)/SUM1	002415
00746	401*	SUM2=C.0	002422
00747	402*	DO 63 J=1,7	002426
00752	403*	63 SUM2=SUM2+(C2(J)-C1(J))/Q	002426
00754	404*	SUM3=1.	002433
00755	405*	SUM4=C.0	002435
00756	406*	DO 55 J=1,7	002441
00761	407*	64 SUM4=SUM4+(TW*RO(1)*C1(J)/(W(J)*SUM1)**1.5)*(1+RT(1)/2./PP1(J)/	002442
00761	408*	1 REYN*(U(1)-4.*V(2)/Q+V(3)/Q)+(3.1416*GAMMA*W(J)*SUM1*TF5/2./T(1)/	002442
00761	409*	2STAGFS)**0.5)/C1(J)*(2.-AC)/AC*AMACH/REYN*EL(J)/PR*(W(J)*SUM1)*	002442
00761	410*	3RT(1)/RO(1)*SUM2)	002442
00763	411*	SUM5=C.0	002514
00764	412*	DO 66 J=1,7	002520
00767	413*	66 SUM5=SUM5+C1(J)*RO(1)/((W(J)*SUM1)**1.5)*(1+RT(1)/2./PP1(J)/REYN	002520
00767	414*	1*(U(1)-4.*V(2)/Q+V(3)/Q))	002520
00771	415*	USLIP=RT(1)/P(1)/REYN/AMACH*((6.2832*STAGFS*T(1)/GAMMA/TFS)**0.5)*	002544
00771	416*	1*(2.-AC)/AC/2.*(4.*U(2)-3.*U(1)-U(3))/2./Q	002544
00772	417*	TSLIP=(2.-AC)/AC*(3.1416**0.5)	002600
00773	418*	TSLIP=TSLIP*(AMACH*RT(1)*GAMMA/2./REYN/2./GAMMA-1.)*(GAMMA*TFS/	002604
00773	419*	12./T(1)/STAGFS)**0.5)*(T(2)-T(1))/Q-2.5*P(1)*RT(1)*	002604
00773	420*	*AMACH**3.	002604
00773	421*	2*((GAMMA*TFS/STAGFS)**1.5)/RO(1)/REYN*((2.*T(1))*0.5)*EL1/PR*	002604
00773	422*	3SUM2)*SUM3	002604
00774	423*	TSLIP=(TSLIP+SUM4)/SUM5	002661
00775	424*	USCONT=(USLIP-U(1))/U(1)	002664
00776	425*	USCONS=USCONT	002670
00777	426*	USCONT=ABS(USCONT)	002671
01000	427*	IF(USCONT.GT.EPSI) U(1)=U(1)+USCONS/USCONT*EPSI*U(1)	002673
01002	428*	IF(USCONT.LE.EPSI) U(1)=USLIP	002705
01004	429*	TSCONT=(TSLIP-T(1))/T(1)	002713
01005	430*	TSCONS=TSCONT	002717
01006	431*	TSCONT=ABS(TSCONT)	002720
01007	432*	IF(TSCONT.GT.EPSI) T(1)=T(1)+TSCONS/TSCONT*EPSI*T(1)	002722
01011	433*	IF(TSCONT.LE.EPSI) T(1)=TSLIP	002734
01013	434*	RT(1)=(1.459C-05*(T(1)*STAGFS)**1.5)/((T(1)*STAGFS+110.4)*RTFS)	002742
01013	435*	C CORRECTION TO V PROFILC	002742
01014	436*	DO 70 N=2,NDIV	002762
01017	437*	V2=V(N+1)-2.*V(N)+V(N-1)	002764
01020	438*	P1=P(N+1)-P(N-1)	002771
01021	439*	V1=V(N+1)-V(N-1)	002774
01022	440*	RT1=RT(N+1)-RT(N-1)	002777
01023	441*	U1=U(N+1)-U(N-1)	003002
01023	442*	C SPHERC/CYLINDER OPTION FOLLOWS	003002
01024	443*	IF(XBOD=0.9.GT.0.2) GO TO 67	003005
01024	444*	C SPHERC	003005
01026	445*	CAPV=-RT(N)*V2/Q**2+0.375*REYN/Q*(P1+RO(N)*V(N)*V1)-V1/Q	003007
01026	446*	1*(RT(N)/X(N)+RT1/4./Q)-RT(N)*U1/4./Q/X(N)	003007
01027	447*	CAPV=CAPV+(U(N)+V(N))/X(N)*(3.5*RT(N)/X(N)+RT1/2./Q)	003035

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01030	448*	DERV=2.*RT(N)/Q**2+0.375*REYN/Q*RO(N)*V1+3.5*RT(N)/X(N)**2	00305 2
01030	449*	1+RT1/2./Q/X(N)	003052
01031	450*	GO TO 68	00307 2
01031	451*	CYLINDER	003072
01032	452*	67 CAPV=-RT(N)*V2/Q**2+0.375*REYN/Q*(P1+RO(N)*V(N)*V1)-V1/Q/2.0	00307 4
01032	453*	1*(RT(N)/X(N)+RT1/2.0/Q)-RT(N)*U1/8.0/Q/X(N)	003074
01033	454*	CAPV=CAPV+(U(N)+V(N))/X(N)*(7.0*RT(N)/X(N)+RT1/Q)/4.0	003126
01034	455*	DERV=2.0*RT(N)/Q**2+C.375*REYN/Q*RO(N)*V1+(7.0*RT(N)/X(N)**2	003145
01034	456*	1+RT1/Q/X(N))/4.0	003145
01034	457*	C TO FIND CCCV	003145
01035	458*	68 SIGV=ABS(CPSI*DERV*V(N)/CAPV	003167
01036	459*	IF(SIGV.LT.1.)CCCV=SIGV	003174
01040	460*	IF(SIGV.GE.1.)CCCV=1.	003202
01042	461*	70 V(N)=V(N)-CCCV*CAPV/DERV	003210
01044	462*	V(2)=(V(3)-4.*DELTA*U(1)*EFFR)/4.	003216
01044	463*	C TO FIND CORRECTION TO U PROFILE	003216
01045	464*	GO 80 N=2,NDIV	003224
01050	465*	U2=U(N+1)-2.*U(N)+U(N-1)	003245
01051	466*	U1=U(N+1)-U(N-1)	00325 2
01052	467*	RT1=RT(N+1)-RT(N-1)	003255
01053	468*	V1=V(N+1)-V(N-1)	00326 0
01054	469*	CAPU=-RT(N)*U2/Q**2+REYN*RO(N)*V(N)*U1/2./Q	003263
01054	470*	C SPHERE/CYLINDER OPTION FOLLOWS	00326 3
01055	471*	IF(XBOD-0.3 .GT. C.2) GO TO 71	003274
01055	472*	C SPHERE	00327 4
01057	473*	CAPU=CAPU+(U(N)+V(N))/X(N)*(8.*RT(N)/3./X(N)+RT1/2./Q+REYN*RO(N)	003276
01057	474*	1*U(N))	003276
01060	475*	DERU=2.*RT(N)/Q**2+8.*RT(N)/3./X(N)**2+RT1/2./Q/X(N)+REYN*RO(N)	003314
01060	476*	1*(2.*U(N)+V(N))/X(N)	003314
01061	477*	GO TO 73	003335
01061	478*	C CYLINDER	003335
01062	479*	71 CAPU=CAPU+(U(N)+V(N))/X(N)*(7.*RT(N)/3./X(N)+RT1/2./Q+PEYN*RO(N)	003337
01062	480*	1*U(N))	003337
01063	481*	DERU=2.*RT(N)/Q**2+7.0*RT(N)/3./X(N)**2+RT1/2./Q/X(N)+REYN*RO(N)	003362
01063	482*	1*(2.0*U(N)+V(N))/X(N)	00336 2
01064	483*	73 CAPU=CAPU+RT(N)*V1/6.0/Q/X(N)-U1/Q*(RT(N)/X(N)+RT1/4.0/Q)	003404
01064	484*	1+2.0*P2(N)*REYN/X(N)	003404
01064	485*	C TO FIND CCCU	003404
01065	486*	SIGU=ABS(CPSI*DERU*U(N)/CAPU)	003431
01066	487*	IF(SIGU.LT.1.)CCCU=SIGU	003437
01070	488*	IF(SIGU.GE.1.)CCCU=1.	003445
01072	489*	80 U(N)=U(N)-CCCU*CAPU/DERU	003453
01072	490*	C SPECIES PRODUCTION RATE	00345 3
01074	491*	DO 90 N=1,NDIV1	003461
01077	492*	W5=T(N)*STAGFS	00347 4
01100	493*	W6=UNIR*W5	003476
01101	494*	W7=118000./W6	003500
01102	495*	W8=W7**1.5	003503
01103	496*	W9=224900./W6	003510
01104	497*	W10=150000./W6	003513
01105	498*	W11=W10**2	003516
01106	499*	W12=39100./W6	003520
01107	500*	W13=7080./W6	003523
01110	501*	W14=75500./W6	003526
01111	502*	W15=128500./W6	003531
01112	503*	W16=85520./W6	003534

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01113	504*	W17=63290./W6	003537
01114	505*	W18=W5**0.5	003542
01115	506*	W19=W5**2.5	003547
01116	507*	W20=500./W6	003554
01117	508*	W210=-10./W6	003557
01120	509*	IF(W7.CT.80.)GO TO 81	003562
01122	510*	FK1=2.5*10.**11*W19*W3/EXP(W7)	003566
01123	511*	GO TO 82	003576
01124	512*	31 FK1=C.	003600
01125	517*	32 RK1=2.08*10.**8*W5*W8	003601
01126	514*	IF(W9.CT.80.)GO TO 83	003604
01130	515*	FK2=1.7*10.**12*W18*W9/EXP(W9)	003610
01131	516*	GO TO 34	003620
01132	517*	33 FK2=C.	003622
01133	513*	34 RK2=9.444*10.**10*W18*W9	003623
01134	519*	IF(W10.CT.80.)GO TO 85	003626
01136	520*	FK3=7.*10.**10*W18*W11/EXP(W10)	003632
01137	521*	GO TO 86	003642
01140	522*	35 FK3=C.	003644
01141	523*	80 RK3=1.75*10.**10*W18*W11	003645
01142	524*	IF(W12.CT.80.)GO TO 87	003650
01144	525*	FK4=3.2*10.**9*W5/EXP(W12)	003654
01145	526*	GO TO 89	003663
01146	527*	37 FK4=C.	003665
01147	528*	IF(W13.CT.80.)GO TO 89	003665
01151	529*	93 RK4=13.33*10.**9*W5/EXP(W13)	003672
01152	530*	GO TO 932	003700
01153	531*	881 RK4=C.	003702
01154	532*	382 IF(W14.CT.80.)GO TO 91	003703
01156	533*	FK5=7.*10.**13/EXP(W14)	003706
01157	534*	GO TO 92	003714
01160	535*	91 FK5=C.	003716
01161	536*	IF(W20.CT.80.)GO TO 921	003716
01163	537*	90 RK5=1.55*10.**13/EXP(W20)	003723
01164	538*	GO TO 922	003730
01165	539*	921 RK5=C.	003732
01166	540*	922 IF(W15.CT.80.)GO TO 93	003733
01170	541*	FK6=9.1*10.**24*W19/EXP(W15)	003736
01171	542*	GO TO 34	003745
01172	543*	90 FK6=C.	003747
01173	544*	94 IF(W16.CT.80.)GO TO 95	003750
01175	545*	RK6=4.79*10.**23*W19/EXP(W16)	003753
01176	546*	GO TO 96	003762
01177	547*	95 RK6=C.	003764
01200	548*	36 IF(W17.CT.80.)GO TO 97	003765
01202	549*	FK7=6.4*10.**9*W18/EXP(W17)	003770
01203	550*	GO TO 98	003777
01204	551*	97 FK7=C.	004001
01205	552*	98 RK7=1.79*10.**19*W5/EXP(W210)	004002
01206	553*	W2=RO(N)*ROFS	004010
01207	554*	W3=W2**2	004013
01210	555*	R1=W3*(CN(N)/WTN+CNO(N)/WTN0+2.*CN2(N)/WTN2+9.*C02(N)/WT02+	004015
01210	556*	125.*C0(N)/WT0)	004015
01211	557*	R1=R1*(FK1*C02(N)/WT02-W2*RK1*C0(N)**2/WT0**2)	004041
01212	558*	R2=W3*(C0(N)/WT0+C02(N)/WT02+CNO(N)/WTN0+2.47*CN2(N)/WTN2+2.15*	004055
01212	559*	110.**5*CN(N)/W5/WTN)	004055

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01213	567*	R2=R2*(FK2*CN2(N)/WTN2-W2*RK2*CN(N)**2/WTN**2)	004077
01214	561*	R3=W3*(C02(N)/WT02+CN2(N)/WTN2+20.*(CNO(N)/WTN0+C0(N)/WT0+1*CN(N)/WTN))	004113
01215	563*	R3=R3*(FK3*CNO(N)/WTN0-W2*RK3*CN(N)*C0(N)/WTN/WT0)	004127
01216	554*	R4=W3*(FK4*CNO(N)*C0(N)/WTN0/WT0-RK4*C02(N)*CN(N)/WT02/WTN)	004143
01217	565*	R5=W3*(FK5*CN2(N)*C0(N)/WTN2/WT0-RK5*CNO(N)*CN(N)/WTN0/WTN)	004160
01220	565*	R6=W3*(FK6*CN2(N)*C02(N)/WT02/WTN2-RK6*(CNO(N)/WTN0)**2)	004175
01221	567*	R7=W3*(FK7*CN(N)*C0(N)/WTN/WT0-RK7*(CNOI(N)/WTNOI)**2)	004211
01222	561*	W4=ROFS*UFS/RAD9	004225
01223	569*	W02(N)=WT02*(-R1+R4-R6)/W4	004227
01224	570*	WNC(N)=-WTN2*(R2+R5+R6)/W4	004234
01225	571*	WNC(N)=WTN0*(-R3-R4+R5+2.*R6)/W4	004241
01226	572*	WNC(N)=WTN*(2.*R2+R3+R4+R5-R7)/W4	004251
01227	577*	W0(N)=WT0*(2.*R1+R3-R4-R5-R7)/W4	004262
01230	574*	WNOI(N)=WTNOI*R7/W4	004273
01231	575*	WEL(N)=WTEL*R7/W4	004277
01232	575*	CONTINUE	004310
01232	577*	C CORRECTION TO CONCENTRATION PROFILES	004310
01234	573*	DO 100 N=2,NDIV	004310
01237	579*	RT1=RT(N+1)-RT(N-1)	004315
01240	580*	W21=-RO(N)*V(N)/2./Q	004320
01241	581*	W22=(2.*RT(N)/X(N)+RT1/2./Q)/2./Q/REYN/SC1	004325
01242	582*	W27=RT(N)/Q**2/REYN/SC1	004337
01243	587*	DELC=-2.*RT(N)/REYN/SC1/Q**2	004344
01243	534*	C OXYGEN MOLECULE	004344
01244	535*	CC01=C02(N+1)-C02(N-1)	004351
01245	536*	C073=C02(N+1)-2.*C02(N)+C02(N-1)	004354
01246	587*	CAP02=W02(N)+(W21+W22)*C021+W23*C023	004361
01247	533*	SI02=ABS(EPSI*DERC*C02(N)/CAP02)	004367
01250	589*	IF(SIG02.LT.1.)CCC02=SIG02	004377
01252	590*	IF(SIG02.GE.1.)CCC02=1.	004405
01254	591*	C01(N)=C02(N)-CCC02*CAP02/DERC	004413
01254	592*	C NO ION	004413
01255	593*	W221=(2.*RT(N)/X(N)+RT1/2./Q)/2./Q/REYN/SC2	004420
01256	594*	W231=RT(N)/Q**2/REYN/SC2	004430
01257	595*	DERC1=-2.*RT(N)/REYN/SC2/Q**2	004435
01260	596*	CNOI1=CNOI(N+1)-CNOI(N-1)	004442
01261	597*	CNOI3=CNOI(N+1)-2.*CNOI(N)+CNOI(N-1)	004445
01262	598*	CAPNOI=WNOI(N)+(W21+W221)*CNOI1+W231*CNOI3	004452
01263	599*	SIGNOI=ABS(EPSI*DERC1*CNOI(N)/CAPNOI)	004460
01264	600*	IF(SIGNOI.LT.1.)CCCN0I=SIGNOI	004465
01266	601*	IF(SIGNOI.GE.1.)CCCN0I=1.	004473
01270	602*	CNOI(N)=CNOI(N)-CCCN0I*CAPNOI/DERC1	004501
01270	603*	C NITROGEN ATOM	004501
01271	604*	CN1=CN(N+1)-CN(N-1)	004506
01272	605*	CN3=CN(N+1)-2.*CN(N)+CN(N-1)	004511
01273	606*	CAPN=WN(N)+(W21+W22)*CN1+W23*CN3	004516
01274	607*	SIGN1=ABS(EPSI*DERC1*CN(N)/CAPN)	004523
01275	608*	IF(SIGN1.LT.1.)CCCN=SIGN1	004530
01277	609*	IF(SIGN1.GE.1.)CCCN=1.	004536
01301	610*	CN(N)=CN(N)-CCCN*CAPN/DERC	004544
01301	611*	C OXYGEN ATOM	004544
01302	612*	C01=C0(N+1)-C0(N-1)	004551
01303	613*	C07=C0(N+1)-2.*C0(N)+C0(N-1)	004554
01304	614*	CAP0=W0(N)+(W21+W22)*C01+W23*C03	004561
01305	615*	SIG0=ABS(EPSI*DERC*C0(N)/CAP0)	004566

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